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Effects of molecular diffusion and of thermal expansion on the structure and dynamics of premixed flames in turbulent flows of large scale and low intensity

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To study effects of flow inhomogeneities on the dynamics of laminar flamelets in turbulent flames, with account taken of influences of the gas expansion produced by heat release, a previously developed theory of premixed flames in turbulent flows, that was based on a diffusive-thermal model in which thermal expansion was neglected, and that applied to turbulence having scales large compared with the laminar flame thickness, is extended by eliminating the hypothesis of negligible expansion and by adding the postulate of weak-intensity turbulence. The consideration of thermal expansion motivates the formal introduction of multiple-scale methods, which should be useful in subsequent investigations. Although the hydrodynamic-instability mechanism of Landau is not considered, no restriction is imposed on the density change across the flame front, and the additional transverse convection correspondingly induced by the tilted front is described. By allowing the heat-to-reactant diffusivity ratio to differ slightly from unity, clarification is achieved of effects of phenomena such as flame stretch and the flame-relaxation mechanism traceable to transverse diffusive processes associated with flame-front curvature. By carrying the analysis to second order in the ratio of the laminar flame thickness to the turbulence scale, an equation for evolution of the flame front is derived, containing influences of transverse convection, flame relaxation and stretch. This equation explains anomalies recently observed at low frequencies in experimental data on power spectra of velocity fluctuations in turbulent flames. It also shows that, concerning the diffusive-stability properties of the laminar flame, the density change across the flame thickness produces a shift of the stability limits from those obtained in the purely diffusive-thermal model. At this second order, the turbulent correction to the flame speed involves only the mean area increase produced by wrinkling. The analysis is carried to the fourth order to demonstrate the mean-stretch and mean-curvature effects on the flame speed that occur if the diffusivity ratio differs from unity.

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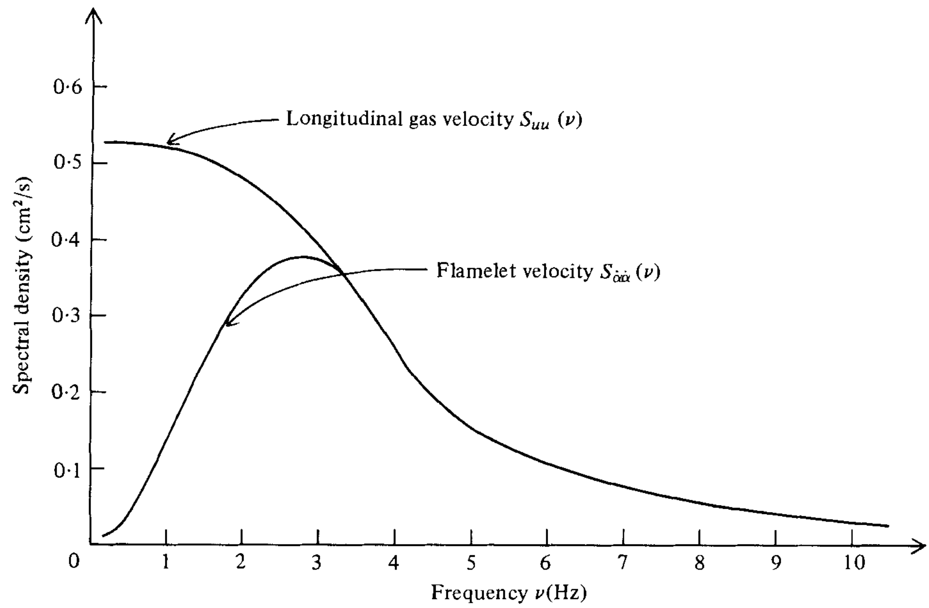

FIGURE 1. Representative power spectra for the longitudinal gas velocity and for the flamelet velocity, measured by Boyer, Clavin & Sabathier (1981).

1. Introduction

In a previously developed theory (Clavin & Williams 1979) for the structure and propagation velocities of premixed flames in turbulent flows having an integral scale L_1 large compared with the laminar flame thickness d, it was found that in a first approximation each laminar flamelet moves in the laboratory frame with the streamwise Eulerian displacement $\hat{\mathbf{a}}$ of the turbulent flow. Although this result helped to clarify the structure of turbulent flames in the limiting regime for which the turbulent flame consists of an ensemble of wrinkled laminar flames, it led to fundamental uncertainties concerning the turbulent flame thickness d_T. It was found that d_T is of the order of the root-mean-square displacement $(\overline{\hat{a}^2})^{\frac{1}{2}}$ when this quantity is large compared with d. The corresponding Lagrangian displacement is not stationary; its mean square increases with time, thereby producing the well-known phenomenon of turbulent diffusion or dispersion. If $\hat{\mathbf{a}}$ behaves similarly, then d_T is predicted to increase with time. In fact, whenever the power spectrum $S_{uu}(\nu)$ of the streamwise Eulerian velocity fluctuation $\hat{u}$ of the turbulent flow approaches a nonzero value in the limit of zero frequency ν, the quantity $(\overline{\hat{a}^2})^{\frac{1}{2}}$ must diverge for large times.

Uncertainties concerning d_T and concerning motion of laminar flamelets in turbulent flows prompted initiation of experimental work on weakly turbulent premixed flames by Sabathier, Boyer & Clavin (1981). Experimental conditions were sought that were as close as possible to those considered in the theoretical work of Clavin & Williams (1979). In addition to measuring $S_{uu}(\nu)$ just ahead of the flame by laser–Doppler techniques, Sabathier *et al.* employed the optical method proposed by Boyer (1980) to measure the power spectrum $S_{\dot{a}\dot{a}}(\nu)$ of the local velocity of the wrinkled flame front for frequencies from 0·1 to 20 Hz. If the flamelets follow the displacement $\hat{\mathbf{a}}$ as predicted, then $S_{uu}(\nu)$ and $S_{\dot{a}\dot{a}}(\nu)$ should coincide. As illustrated schematically in figure 1, agreement is achieved at frequencies above about 3 Hz, but at lower frequencies a premature peak develops in $S_{\dot{\alpha}\dot{\alpha}}(\nu)$ which decreases toward zero while $S_{uu}(\nu)$ is still increasing. This discrepancy precludes the possibility of employing flame data to obtain information on the behaviour of $S_{uu}(\nu)$ in the frequency range of uncertainty, below 10^{-2} Hz, and it poses a new problem of explaining the departure from predictions between 10^{-1} and 3 Hz. The source of the discrepancy is identified in the theoretical analysis presented herein as arising from small differences between molecular diffusivities of heat and reactants.

The experimental conditions correspond to a mean-flow velocity u_T (ahead of the flame) of about 20 cm/s, a turbulence intensity $(\overline{\hat{u}^2})^{\frac{1}{2}}/u_T$ of about 8 % and an integral scale L_1 of about 1 cm. This results in low frequencies ($S_{uu}(\nu) \simeq 0$ unless $\nu < 20$ Hz) and in an observed turbulent-flame thickness d_T that remains constant at about 0·2 cm. The average flame position is oriented perpendicular to the mean flow, as suggested by Williams (1970). Details of the experiment have been described by Sabathier (1980). Since the laminar flame thickness d is about 0·05 cm, the parameter $\epsilon = d/L_1$ is small, consistent with the gradient expansion proposed by Clavin (1979) and employed by Clavin & Williams (1979). Under these conditions, the experimental results shown in figure 1 may be described by stating that the flame behaves as a high-pass filter. This is unusual in physical systems; the flamelets might be expected to follow the gas motion at low frequencies but to lag at high frequencies, resulting in behaviour typical of a low-pass filter, with the power spectra agreeing at low frequencies but not at high frequencies.

The phenomena operative to produce the observed behaviour clearly cannot be simple lag processes. It is possible to estimate the response time of a laminar flamelet as the transit time t_L of a fluid element through the laminar flame. Since $t_L = d/u_L$, where the laminar flame speed u_L is about 20 cm/s, the response time is roughly $2{\cdot}5 \times 10^{-3}$ s, which corresponds to a frequency between 10^2 and 10^3 Hz. Thus, flamelet lag may be expected to occur only at frequencies much higher than those of figure 1.

That the explanation may reside in diffusive-thermal effects may be inferred from results of Barenblatt, Zel'dovich & Istratov (1962), who show that the characteristic time of relaxation t_R associated with transverse diffusion mechanisms that govern laminar flame stability, the self-evolution time, is roughly Λ^2/D_{th}, where Λ is a representative wavelength of the wrinkles and where D_{th}, the ratio of the thermal conductivity to the product of the density with the specific heat at constant pressure, is the thermal diffusivity of the gas mixture. In the flame typically $D_{th} \simeq 1$ cm²/s, and Λ may be expected to be of the order of L_1, about 1 cm. Hence t_R is roughly of the order of 1 s, corresponding to a frequency of 1 Hz. This value lies in the centre of the range of frequencies over which the spectra in figure 1 disagree. Thus it is consistent to assume that the transverse diffusive mechanism of relaxation is responsible for the discrepancy in the observed spectra. In this view, the mechanism involves the self-evolution of the flame front, wrinkled without time delay by the turbulent flow. When the characteristic evolution time t_T of the turbulent flow is much shorter than t_R, the transverse diffusion mechanism is hidden by the forced movement of the front induced by the turbulent flow, but when $t_T > t_R$, corresponding to the low-frequency range in figure 1, $\nu < t_R^{-1}$, the self-evolutionary movement of the wrinkled front cannot be neglected and in the stable regime tends to reduce the motion of the front in comparison with the driving motion of the turbulence.

The theory developed in the present paper indicates that this transverse diffusive mechanism of relaxation remains efficient at low frequencies even when the convective phenomena produced by the expansion of gases through the flame thickness are taken into account. This theory also provides quantitative predictions of the phenomenon just described, in qualitative agreement with the recent experimental data of Boyer, Clavin & Sabathier (1981).

2. Formulation

The diffusive-thermal model of flame motion as originally proposed by Barenblatt, Zel'dovich & Istratov (1962) excluded density variations, thereby uncoupling the continuity and momentum-conservation equations from the equations of species and energy conservation, which then govern the flame dynamics. This serves the useful purpose of retaining the diffusive effects while eliminating hydrodynamic instability (Landau 1944), which perhaps does not play an important role in experiments. Recently, extensive development of this diffusive-thermal model has been based on asymptotic expansions for large values of the non-dimensional activation temperature and has led to clarification of diffusive-instability mechanisms of premixed flames (Sivashinsky 1977*a*; Joulin & Clavin 1979; Joulin 1979; García-Ybarra & Clavin 1981). Some mathematical justification for the adoption of this model has been given by Matkowsky & Sivashinsky (1979), who analysed the double limit, $\beta \equiv T_a(T_f-T_0)/T_f^2 \rightarrow \infty$ and $\gamma \equiv (T_f-T_0)/T_f \rightarrow 0$, where T_a is the activation temperature, T_f the adiabatic flame temperature and T_0 the initial temperature of the fresh mixture. The non-dimensional activation energy β and the non-dimensional heat release γ are significant parameters in flame propagation. The turbulent-flame studies of Clavin & Williams (1979, 1981) also concern the limit $\beta \rightarrow \infty$, $\gamma \rightarrow 0$. In the present work, the realistic limit $\beta \rightarrow \infty$ is retained, but the troubling approximation $\gamma \rightarrow 0$ is no longer necessary because of simplifications that result from the restriction to low turbulence intensities. On purpose, effects of hydrodynamic instability are still not investigated; i.e. far-field modifications of the flow, produced by flame movement, are not considered. The results concerning flame behaviour differ quantitatively but not qualitatively from those obtained by a parallel analysis in which $\gamma \rightarrow 0$.

More-detailed discussion of influences of having $\gamma \neq 0$ seems warranted here. Gas expansion that occurs through the wrinkled laminar flame when γ is non-zero produces transverse convection effects, beyond those present when $\gamma = 0$, that modify the mass and heat balances within the thickness of the turbulent flame. These influences, and the associated modifications to the turbulent velocity field within and across the turbulent flame, are the main differences obtained herein from results for the limit $\gamma \rightarrow 0$. In the present work, the velocity field just ahead of the wrinkled flame is considered to be given. Landau (1944) showed that when $\gamma \neq 0$ incompressible effects in the far field ahead of and behind the flame modify the velocity field ahead of the flame front. Sivashinsky (1977*b*) gave an equation for evolution of a flame front in a uniform flow for small γ, including a proposed form for this hydrodynamical modification. Although such modifications are not considered herein, the present formulation may provide a starting point for improved analyses of the hydrodynamical phenomena. The approach would be to include the associated far-field modifications to the velocity field just ahead of the wrinkled flame. Such an analysis is performed by Pelcé & Clavin (1982) for the stability of planar fronts. However, even in the absence of this extension of the theory, the present analysis provides relationships among experimentally measurable quantities, such as those shown in figure 1 corresponding to stable fronts.

Following Clavin & Williams (1979, 1981), attention will continue to be restricted to exothermic reactions amenable to a one-step approximation, with the degree of progress describable in terms of the mass fraction Y_A of a single reactant, taken to be a limiting or deficient reactant in the sense that $Y_A = 0$ defines completion of the reaction. An Arrhenius rate is adopted in the generalized sense described by Clavin & Williams (1979), such that as $\beta \to \infty$ the chemistry is confined to a fluctuating surface, termed the flame sheet or reaction zone, whose position is employed to define the origin of a moving co-ordinate system. With the density ρ variable, the mass-based thermal diffusivity ρD_{th} is assumed to be constant and is used in conjunction with the constant upstream density ρ_0 and the mass burning rate $\rho_0 u_L$ of the steady, planar, laminar flame for forming non-dimensional space and time variables x, y, z and t. Thus, $d = \rho D_{th}/\rho_0 u_L$ and $t_L = \rho D_{th}/\rho_0 u_L^2$ are the units of length and time, respectively. The co-ordinates are selected so that the turbulence will be stationary in t and homogeneous in y and z, with the average flow in the positive x-direction. The location of the flame sheet will be defined by $x = \alpha(y,z,t)$, and the moving co-ordinates $\xi = x-\alpha$, $\eta = y$, $\zeta = z$, $\tau = t$ are introduced. Subscript notation will be employed for partial derivatives of α.

The conservation equations to be written in the moving co-ordinates are those for mass, momentum, energy and reactant. These involve the components of velocity in the (x,y,z)-co-ordinate system, denoted by U, v and w, respectively, after non-dimensionalization through division by u_L. Let $r = \rho/\rho_0$ be the density ratio, $\mathbf{v} = (v,w)$ represent the transverse velocity, ∇ denote the transverse gradient (involving differentiation with respect to y and z) and

$$s = r(U-\alpha_t-\mathbf{v}\,.\,\nabla\alpha) \tag{1}$$

identify the non-dimensional longitudinal mass flux in the moving frame. Then continuity becomes

$$\partial r/\partial\tau + \partial s/\partial\xi + \partial(rv)/\partial\eta + \partial(rw)/\partial\zeta = 0. \tag{2}$$

Conservation of momentum is employed under the assumptions that the coefficient of shear viscosity μ and the coefficient of bulk viscosity κ are constant. Constant Prandtl numbers, based on the first and second coefficients of viscosity, respectively, may then be defined as $P = \mu/\rho D_{th}$ and $P' = (\frac{1}{3}\mu+\kappa)/\rho D_{th}$. A constant representative Mach number is $M = (\rho_0 u_L^2/p_0)^{\frac{1}{2}}$, where the constant pressure p_0 is the mean pressure far upstream. The non-dimensional pressure, the ratio of the pressure to p_0, will be denoted by $1+M^2p$. In the moving frame the longitudinal component of momentum conservation then becomes

$$r\frac{\partial U}{\partial\tau} + (s+P\nabla^2\alpha)\frac{\partial U}{\partial\xi} + rv\frac{\partial U}{\partial\eta} + rw\frac{\partial U}{\partial\zeta} = -\frac{\partial p}{\partial\xi} + P\Delta' U + P'\frac{\partial}{\partial\xi}\left[\frac{\partial(s/r)}{\partial\xi} + \frac{\partial v}{\partial\eta} + \frac{\partial w}{\partial\zeta}\right], \tag{3}$$

where

$$\Delta' \equiv (1+|\nabla\alpha|^2)\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2} - 2\alpha_y\frac{\partial^2}{\partial\xi\,\partial\eta} - 2\alpha_z\frac{\partial^2}{\partial\xi\,\partial\zeta} \tag{4}$$

represents the three-dimensional Laplacian with the term $-(\nabla^2\alpha)\,\partial/\partial\xi$ removed. One of the transverse components of momentum conservation is

$$r\frac{\partial v}{\partial \tau}+(s+P\nabla^2\alpha)\frac{\partial v}{\partial \xi}+rv\frac{\partial v}{\partial \eta}+rw\frac{\partial v}{\partial \zeta}$$
$$=-\frac{\partial p}{\partial \eta}+\alpha_v\frac{\partial p}{\partial \xi}+P\Delta' v+P'\left(\frac{\partial}{\partial \eta}-\alpha_v\frac{\partial}{\partial \xi}\right)\left[\frac{\partial(s/r)}{\partial \xi}+\frac{\partial v}{\partial \eta}+\frac{\partial w}{\partial \zeta}\right]. \quad (5)$$

The other is similar.

Energy conservation is written subject to the assumption that M is small. The non-dimensional enthalpy or temperature variable Θ is defined as the departure of the thermal enthalpy from its inlet value, divided by the difference between the thermal enthalpy of the fully burnt gas and that of the fresh mixture. In the moving frame, conservation of energy then becomes

$$r\frac{\partial \Theta}{\partial \tau}+(s+\nabla^2\alpha)\frac{\partial \Theta}{\partial \xi}+rv\frac{\partial \Theta}{\partial \eta}+rw\frac{\partial \Theta}{\partial \zeta}=\Delta'\Theta+\Lambda_{\mathrm{L}}F(\Theta,\Psi), \quad (6)$$

in which the last term describes the chemical production of heat. The variable Ψ in the function F is the normalized reactant concentration Y_A/Y_{A0}, where Y_{A0} is the value of Y_A in the gas far upstream. The Arrhenius form of F for a reaction of order n occurring in a mixture with constant specific heat is

$$F(\Theta,\Psi)=\beta^{n+1}\Psi^n\exp\{-\beta(1+\Theta)/[1-\gamma(1-\Theta)]\}, \quad (7)$$

where γ is the heat-release parameter defined previously ($\gamma<1$). The factor Λ_{L} is the constant laminar burning-rate eigenvalue, a number of order unity related to a Damköhler number and defined in equation (4) of Clavin & Williams (1979). When $F(\Theta,\Psi)$ is given by (7), it is assumed that the pre-exponential factor in the rate expression has a suitable, weak, temperature and concentration dependence to make Λ_{L} constant. The equation of state relates r to p, Θ and Ψ. When M is small and the specific heat and mean molecular weight are constant, this equation yields

$$r=[1+\Theta\gamma/(1-\gamma)]^{-1}. \quad (8)$$

Equations (7) and (8) will be employed herein to present explicit results in their simplest form, although it should be evident that the methods are not restricted to these particular rate and state formulas.

In the equation for reactant conservation it is assumed that the Lewis number $L=D_{\mathrm{th}}/D_A$ is constant. Here D_A is the diffusion coefficient for species A in the mixture, this species being assumed to obey Fick's law. In the moving co-ordinates, species conservation is

$$r\frac{\partial \Psi}{\partial \tau}+(s+L^{-1}\nabla^2\alpha)\frac{\partial \Psi}{\partial \xi}+rv\frac{\partial \Psi}{\partial \eta}+rw\frac{\partial \Psi}{\partial \zeta}=L^{-1}\Delta'\Psi-\Lambda_{\mathrm{L}}F(\Theta,\Psi). \quad (9)$$

In addition to the previously stated conditions of stationarity and homogeneity, there are boundary conditions at $\xi=-\infty$ (e.g. $\Theta=0$, $\Psi=1$) and at $\xi=\infty$ (e.g. $\Theta=1,\Psi=0$). With specified turbulent fluctuations of velocity upstream, the problem is expected to possess a solution only for a particular value of the parameter $m=u_{\mathrm{T}}/u_{\mathrm{L}}$, where u_{T} is the turbulent flame speed. In terms of quantities defined previously, m is the average value of U or of rU for $\xi \to -\infty$. The primary objective will be to investigate properties of the function α and to find m. The diffusive-thermal model ($\gamma \to 0$) of turbulent combustion employed by Clavin & Williams (1979, 1981) is concerned with (6) and (9), with r and s treated as constants. In the present approach, although γ is not small, attention is focused on flows in which r remains constant in the upstream turbulence.

3. Analytical approach

As $\beta \to \infty$, F becomes negligible except in a narrow reaction zone centred at $\xi = 0$. Bush & Fendell (1970) showed how to handle the nonlinearity of F by means of an asymptotic expansion for $\beta \to \infty$; the approach was reviewed by Williams (1971). Clavin & Williams (1979) employed such an approach for turbulent flames to remove the nonlinearity associated with F. The same type of method is adopted herein, with the first two terms of the expansion in β^{-1} retained.

Moreover, the expansion $L = 1 + l/\beta$ is introduced, with l assumed to be of order unity (cf. Joulin & Clavin 1979; Joulin 1979). Reasons for this restriction on the Lewis number have been discussed by Clavin & Williams (1981). An alternative view of the desirability of taking l of order unity, to address the problem identified in §1, is provided by Sivashinsky's (1977*a*) first detailed analysis of laminar-flame stability. On the basis of a diffusive-thermal model he showed that, with the non-dimensionalizations $\tau_R = t_R/t_L$ and $\lambda = \Lambda/d$, the characteristic non-dimensional relaxation time for transverse disturbances is

$$\tau_R = [1 + \tfrac{1}{2}(L-1)\beta]^{-1}\lambda^2. \tag{10}$$

Thus, $\beta(L-1)$, i.e. l, controls the self-evolution mechanisms of the flame front, and to describe these mechanisms properly in an asymptotic development for $\beta \to \infty$ it is necessary to take l of order unity. In real gas mixtures L is near unity, nearer to unity than is P.

According to (10), τ_R is negative for $l < -2$; this corresponds to a regime of diffusive-thermal instability. It will be shown herein that for $\gamma \neq 0$ effects of thermal expansion cause the critical value of l below which diffusive-thermal instability occurs to lie appreciably below -2. Denoting this critical value by l^*, we introduce the further restriction $l > l^*$, so that the wrinkled flames considered will be stable diffusively to transverse disturbances. This condition holds for the particular combustible mixture to which figure 1 applies; the experimental results differ in the unstable regime, identified by Pelcé & Clavin (1982), and these results require further study.

Since the perturbation approach adopted here results in $u_T - u_L \ll u_L$, the generally observed property $L_1/t_T = O(u_T)$ of turbulent flows may be introduced into the present analysis by selecting the ordering $L_1/t_T = O(u_L)$. This weak version of Taylor's hypothesis implies that $t_L/t_T = O(\epsilon)$. Since $d/L_1 = \epsilon$, this selection means that the large-scale turbulence under consideration also is slowly varying when measured on a time scale characteristic of the transit time through a laminar flame. The space and time variables appropriate to the upstream turbulence are $X = \epsilon x$, $Y = \epsilon y$, $Z = \epsilon z$ and $T = \epsilon t$. In anticipation that these properties of the upstream turbulence are reflected in the motion of the reaction zone, the function α will be written as

$\alpha(Y, Z, T)$, where the notation indicates that α is assumed to vary by relative amounts of order unity only over the long space and time scales L_1 and t_T.

Equation (10) suggests that to observe self-relaxation effects it is desirable to investigate changes on time scales large compared with t_T. Specifically, with Λ of order L_1, λ is of order $1/\epsilon$, and therefore (10) implies that τ_R is $O(1/\epsilon^2)$. Hence, in addition to turbulent changes on a non-dimensional time scale such that ϵt is $O(1)$, self-evolutionary changes, on a non-dimensional time scale such that $\epsilon^2 t$ is $O(1)$, may be expected to occur. This suggests that a two-time approach is relevant, with α being taken to depend on four variables, $\alpha = \alpha(Y, Z, T, \epsilon^2 t)$. Although a two-time analysis can produce the results to be derived herein, variations on the longer time scale $\epsilon^{-2} t_L$ are simple enough to be obtained from a one-time analysis by carrying the analysis to higher order in ϵ and then resumming. Therefore, for simplicity in presentation, a one-time approach is employed, and $\alpha(Y, Z, T)$ is sought.

As $\gamma \to 0$, U approaches $m + u$, where u is the non-dimensional longitudinal velocity fluctuation of the upstream flow, having a time-average or ensemble-average value $\bar{u}$ of zero. The result obtained by Clavin & Williams (1979) to lowest order in ϵ may be written as $\alpha_0 = \int u\,dt \equiv a$. This result remains valid here for the dominant term as $\gamma \to 0$ and dictates an ordering for the velocity field. Specifically, to retain the relaxation effects described by (10), in such a manner that they are not hidden by geometrical effects of flame wrinkling, it is desirable to consider situations in which α is $O(1)$; i.e. in which flame displacements are of the same order of magnitude as the laminar-flame thickness. Since $t = T/\epsilon$, this implies that u is $O(\epsilon)$; thus attention is to be focused on weak turbulence.

This reasoning may be extended to conditions under which $\gamma \neq 0$. As a consequence of gas expansion, $\bar{U}$ is no longer constant. Across the wrinkled laminar flame, U must change by an amount of order unity. This change will be taken into account by a function $u_0(\xi)$ that approaches m_0, the first approximation to m, as ξ approaches $-\infty$. Weak turbulence will be described by allowing U to differ from $u_0(\xi)$ by an amount of order ϵ.

The corrections of order ϵ may be expressed in a variety of different ways. A convenient way may be discussed by excluding hydrodynamic instabilities at first and considering the fluctuating velocity field that would be present if there were no flame. Denote the non-dimensional streamwise component of this fluctuating velocity by $\epsilon u_{-\infty}(X, Y, Z, T)$, where $u_{-\infty}$ is taken to be $O(1)$. The average taken with X fixed is $\bar{u}_{-\infty} = 0$, but $\overline{u^2_{-\infty}}$ decreases with increasing X through decay of turbulence, over a very long distance scale such that the change in x is $O(1/\epsilon^2)$. Changes taking place on this scale of decay are not considered herein. The result of Clavin & Williams (1979), to lowest order in ϵ, with $\gamma \to 0$, may equally well be written as

$$\alpha_0(Y, Z, T) = \int u_{-\infty}(0, Y, Z, T)\,dT = a.$$

The presence of the flame will modify the fluctuating velocity further. The correction to the formula $U = u_0 + \epsilon u_{-\infty}$ will be assumed to be $O(\epsilon)$ as a consequence of weak wrinkling and small intensity, and it will be written as $\epsilon u_1 + \epsilon^2 u_2 + \dots$, where $u_1, u_2, \dots$ are $O(1)$. It is relevant to consider over what distance scale in the streamwise direction $u_1, u_2, \dots$ may vary. Variations are anticipated on a scale such that changes in ξ are $O(1)$, as a consequence of passage of the turbulence through the flame. In addition, variations on a longer scale such that changes in $\epsilon\xi$ are $O(1)$ may occur as a consequence of interaction of the flame-induced modifications with the turbulent field. To indicate this two-scale dependence, u_1 will be written as $u_1(\xi, \Xi, Y, Z, T)$, where $\Xi = \epsilon\xi$. When incompressible effects in the incoming gas flow produced by the wrinkling of the front are neglected, to the order to which the analysis is carried, it is found that variations over the larger longitudinal scale occur only downstream from the wrinkled flame. Therefore, although the method of multiple scales is needed in general in analysing u_1, upstream from the flame there is no change on the longer distance scale in the present analysis.

The absence of upstream variations of u_1 on a scale such that changes in Ξ are $O(1)$ implies that if Ξ is set equal to zero in the variable $X = \epsilon(\xi+\alpha) = \Xi+\epsilon\alpha$, then $\epsilon u_{-\infty}(X, Y, Z, T)$ represents the non-dimensional fluctuation of longitudinal velocity just ahead of the wrinkled flame. Therefore $u_j(\xi, \Xi, Y, Z, T) = m_j$ at $\xi = -\infty$ is to be imposed as a boundary condition on u_j $(j = 1, 2, \ldots)$, where the constants m_j in the expansion $m = m_0+\epsilon m_1+\epsilon^2 m_2+\ldots$ of the non-dimensional turbulent flame speed are to be determined in the course of the analysis. As indicated above, $u_{-\infty}$ may be treated as given, in the absence of hydrodynamical effects. If hydrodynamical phenomena occur, then, at $\Xi = 0$, $u_{-\infty}$ is still to be interpreted as the non-dimensional fluctuation of longitudinal velocity just ahead of the wrinkled flame, with the same boundary condition on u_j being imposed, but characteristics of $u_{-\infty}$ are no longer known from properties of non-reacting grid turbulence. Since the dispersion relation of Landau (1944), analogous to (10), shows that the non-dimensional hydrodynamical time t_H/t_L is proportional to λ, with attention restricted to time scales such that T is of order unity the hydrodynamical phenomena may be expected to occur on distance scales such that Ξ is $O(1)$. As stated in § 2, although these hydrodynamical aspects are not analysed herein, the analysis is consistent irrespective of their presence, $u_{-\infty}$ being treated as given but possibly containing hydrodynamical modifications which can be investigated in further studies along the lines of the recent work of Pelcé & Clavin (1982).

An alternative way to express corrections to U of order ϵ would be, for example, to treat the fluctuating velocity just downstream from the wrinkled flame as given. Although there would be no logical inconsistency in adopting this alternative or others, the greater ease of measuring velocity fluctuations upstream and their closer relationship to properties of non-reacting grid turbulence suggest that the approach adopted herein may be more convenient.

Although in many problems properties will vary importantly on both longitudinal distance scales, the results obtained herein may be derived without making use of the method of multiple scales. This is demonstrated in appendix B, where an analysis paralleling that of Clavin & Williams (1979, 1981) is outlined. Often developments are less time-consuming if multiple-scale methods can be avoided. However, for the present problem complexities of the two-scale approach at lower orders in ϵ are overbalanced by relative simplifications that arise at higher orders. Therefore, overall the two-scale method is not appreciably more complicated for the present purpose, even though it involves more elaborate notation and a greater number of equations. Partially because of this simplification, but more importantly because the method establishes the basis for further analyses of phenomena such as inviscid effects in the far field and higher-order effects of flame curvature on flame-front evolution, the two-scale method will be presented in the main text. Through the variable Ξ this method provides a resummation of the expansion in ϵ that may lead systematically to a nonlinear equation for evolution of the flame front.

The specific expansions introduced are as follows:

$$\left.\begin{aligned}
\alpha &= \alpha_0(Y,Z,T)+\epsilon\alpha_1(Y,Z,T)+\epsilon^2\alpha_2(Y,Z,T)+\dots,\\
U &= u_0(\xi,\Xi,Y,Z,T)+\epsilon u_{-\infty}(X,Y,Z,T)+\epsilon u_1(\xi,\Xi,Y,Z,T)+\dots,\\
\mathbf{v} &= \epsilon\mathbf{v}_{-\infty}(X,Y,Z,T)+\epsilon\mathbf{v}_1(\xi,\Xi,Y,Z,T)+\dots,\\
p &= p_0(\xi,\Xi,Y,Z,T)+\epsilon p_{-\infty}(X,Y,Z,T)+\epsilon p_1(\xi,\Xi,Y,Z,T)+\dots,\\
\Theta &= \Theta_0(\xi,\Xi,Y,Z,T)+\epsilon\Theta_1(\xi,\Xi,Y,Z,T)+\dots,\\
\Psi &= \Psi_0(\xi,\Xi,Y,Z,T)+\epsilon\Psi_1(\xi,\Xi,Y,Z,T)+\dots,\\
m &= m_0+\epsilon m_1+\epsilon^2 m_2+\dots.
\end{aligned}\right\} \qquad (11)$$

Corresponding expansions for r and s ($s = s_0+\epsilon s_1+\dots$) are implied by (1) and (8). Here for $\mathbf{v}$ and p the subscript $-\infty$ again identifies properties of the turbulence in the approaching flow upstream from the flame; these properties are treated as being given in advance, so that boundary conditions to be imposed on $\mathbf{v}_j$ and p_j ($j = 1, 2, \dots$) are that these quantities vanish as ξ approaches $-\infty$. The two-scale behaviour of $\mathbf{v}_1$ and p_1 is similar to that of u_1.

The scaling defined here is well suited to the experiment discussed in § 1. It has been indicated that $\epsilon = d/L_1 \simeq 5\times 10^{-2}$ and that $|u| \simeq 8\times 10^{-2}$, so that u is small and $O(\epsilon)$; also, $d_T/d \simeq 4$ experimentally, so that α is $O(1)$. It is possible to introduce different scaling that allows α to be larger, thereby achieving improved accuracy for nonlinear geometrical effects of flame wrinkling while hiding effects of transverse diffusion to the orders investigated. One of these alternative scalings is developed in appendix A, and it results in

$$\left.\begin{aligned}
m &= \overline{[1+|\nabla\alpha|^2]^{\frac{1}{2}}},\\
\alpha_t &= u|_{x=\alpha}-\mathbf{v}|_{x=\alpha}\cdot\nabla\alpha+m-[1+|\nabla\alpha|^2]^{\frac{1}{2}},
\end{aligned}\right\} \qquad (12)$$

which provides the turbulent flame speed and the flame-sheet motion for arbitrary turbulence intensities under conditions such that gradients are sufficiently weak for the local structure of the wrinkled flame to coincide with that of a planar laminar flame and for the local normal to the flame sheet not to approach a perpendicular to the mean-flow direction. In (12), $u|_{x=\alpha}$ and $\mathbf{v}|_{x=\alpha}$ denote the fluctuating parts of the longitudinal- and transverse-velocity fields, respectively, evaluated just upstream from the wrinkled flame, as indicated in appendix A. The geometrical effects contained in the results derived in the main body of the paper correspond to an expansion of (12) for small values of $\nabla\alpha$.

Although scalings other than that in (11) can lead to the same results that are derived herein, the present selection simplifies the derivation of these results, especially concerning the transverse diffusive effects that involve the flame curvature $\nabla^2\alpha$, a quantity that is seen to be $O(\epsilon^2)$. It may be inferred from (10) that these effects are important for flame relaxation and therefore for explaining the spectral discrepancy in figure 1. With the present scaling it is relatively easy to carry the analysis to order ϵ^2 to obtain these effects. Other scalings make it more difficult to retain such effects. An alternative development by Clavin & Williams (1981) did not introduce (11) and contained no special restrictions concerning either the turbulent intensity or the characteristic time of the turbulence. With this greater generality, only the averaged equations could be analysed conveniently at order ϵ^2, and transverse diffusive effects were not revealed. Since the approach of Clavin & Williams (1981) permits turbulent variations on non-dimensional time scales short compared with T, it can be useful for studying the high-frequency lags of flame response that occur on a characteristic time t_L and that are hidden by the scalings adopted herein. However, it is not well-suited for investigation of the self-relaxation phenomena at low frequencies. Therefore, in a sense, the two approaches are complementary.

4. Multiscale analysis

An asymptotic expansion for $\beta \to \infty$ is performed prior to the development in ϵ. As is now well-known (e.g. Joulin & Clavin 1979; Clavin & Williams 1981), this procedure involves introducing an inner reaction zone, having a stretched variable $\beta\xi$, and integrating the conservation equations across this zone to obtain through matching the jump conditions that are applied at $\xi = 0$ in analysing reaction-free forms of the conservation equations on a distance scale such that ξ is $O(1)$. Since the analysis of the inner zone is lengthy, and parallels exactly that of Joulin & Clavin (1979), only the essential results are quoted here. Superscripts will identify orders of the expansion in β^{-1}, e.g. $\Theta = \Theta^{(0)} + \beta^{-1}\Theta^{(1)} + \dots$. When the order of magnitude of transverse gradients is less than β, the reaction zone is quasiplanar and quasistatic, and the jump conditions obtained are

$$\left.\begin{aligned}
&\Theta^{(0)}|_{\xi=0+} = \Theta^{(0)}|_{\xi=0-} = 1, \quad \Theta^{(1)}|_{\xi=0+} = \Theta^{(1)}|_{\xi=0-} \equiv \sigma,\\
&\Psi^{(0)}|_{\xi=0+} = \Psi^{(0)}|_{\xi=0-} = 0, \quad \Psi^{(1)}|_{\xi=0+} = \Psi^{(1)}|_{\xi=0-} = 0,\\
&\partial\Theta^{(0)}/\partial\xi|_{\xi=0-} = -\partial\Psi^{(0)}/\partial\xi|_{\xi=0-} = (1+|\nabla\alpha|^2)^{-\frac{1}{2}}\, e^{\frac{1}{2}\sigma},\\
&\partial\Theta^{(1)}/\partial\xi|_{\xi=0-} + \partial\Psi^{(1)}/\partial\xi|_{\xi=0-} - l\,\partial\Psi^{(0)}/\partial\xi|_{\xi=0-} = \partial\Theta^{(1)}/\partial\xi|_{\xi=0+}.
\end{aligned}\right\} \tag{13}$$

It may be noted that in (13) the new variable $\sigma = \beta(T|_{\xi=0} - T_f)/(T_f - T_0)$ has been introduced, measuring the departure of the local, instantaneous flame temperature from the adiabatic flame temperature; σ of course possess an ϵ-expansion of the same form as α.

Through (8), the conditions given here for Θ imply corresponding jump conditions for r. Since (2) shows that s and its ξ-derivative are continuous, use of (1) results in jump conditions for U. Since (3) and (5) do not contain the chemical production term F, the jump conditions relating p and $\mathbf{v}$ to U are obtained simply by integration over the inner zone; it is found that

$$\left.\begin{aligned}
p|_{\xi=0+} - p|_{\xi=0-} &= P(1+|\nabla\alpha|^2)\left[\left.\frac{\partial U}{\partial\xi}\right|_{\xi=0+} - \left.\frac{\partial U}{\partial\xi}\right|_{\xi=0-}\right] + P'\left[\left.\frac{\partial(s/r)}{\partial\xi}\right|_{\xi=0+} - \left.\frac{\partial(s/r)}{\partial\xi}\right|_{\xi=0-}\right],\\
(\nabla\alpha)&\left[\left.\frac{\partial U}{\partial\xi}\right|_{\xi=0-} - \left.\frac{\partial U}{\partial\xi}\right|_{\xi=0+}\right] = \left.\frac{\partial\mathbf{v}}{\partial\xi}\right|_{\xi=0+} - \left.\frac{\partial\mathbf{v}}{\partial\xi}\right|_{\xi=0-}.
\end{aligned}\right\} \tag{14}$$

The multiscale problem addressed is that of finding the outer solutions to (6) and (9), in the regions $\xi < 0$ and $\xi > 0$ where $F = 0$, satisfying the jump conditions (13), as expansions in ϵ with ξ and Ξ treated as independent variables. Because of the effects of gas expansion it is necessary to treat simultaneously the associated fluid-mechanical problem defined by (2), (3) and (5) with the jump conditions (14). For brevity, the only aspects of the fluid-mechanical problem investigated here are those needed in solving (6) and (9).

Outside the reaction zone, to lowest order in ϵ (1)–(9) yield the relationships

$$\left.\begin{aligned} &\frac{\partial s_0}{\partial \xi} = 0, \quad s_0\frac{\partial \Theta_0}{\partial \xi} = \frac{\partial^2 \Theta_0}{\partial \xi^2}, \quad s_0\frac{\partial \Psi_0}{\partial \xi} = L^{-1}\frac{\partial^2 \Psi_0}{\partial \xi^2}, \\ &r_0 = \left[1+\frac{\Theta_0\gamma}{(1-\gamma)}\right]^{-1}, \quad s_0 = r_0 u_0, \quad s_0\frac{\partial u_0}{\partial \xi} = -\frac{\partial p_0}{\partial \xi}+(P+P')\frac{\partial^2 u_0}{\partial \xi^2}, \end{aligned}\right\} \tag{15}$$

written here in the order that they will be used. Making use of the isothermal, constant-composition conditions in the fresh mixture just ahead of the flame and of boundedness for Θ_0 and Ψ_0 in the burnt gas, we find the solutions to the first three equations in (15):

$$\left.\begin{aligned} &s_0 = S_0(\Xi), \\ &\Theta_0 = \begin{cases} e^{\xi S_0(\Xi)} C_0(\Xi) & (\xi < 0), \\ C_0(\Xi) & (\xi > 0), \end{cases} \\ &\Psi_0 = \begin{cases} 1-e^{L\xi S_0(\Xi)}[1-B_0(\Xi)] & (\xi < 0), \\ B_0(\Xi) & (\xi > 0). \end{cases} \end{aligned}\right\} \tag{16}$$

The functions r_0 and u_0 are obtained from the fourth and fifth formulae in (15) by use of (16). The coefficients appearing in (16) may be functions of Ξ. They also may depend upon Y, Z and T, but for brevity of notation these latter dependences are not exhibited explicitly in the present section. The dependence on Ξ cannot be determined until the following order in ϵ. However, the first two lines of the jump conditions in (13) provide the relationships $C_0^{(0)}(0) = 1$, $B_0^{(0)}(0) = 0$ and $B_0^{(1)}(0) = 0$; the last line then gives $C_0^{(1)}(0) = 0$, and the third line then shows that $S_0^{(0)}(0) = 1$.

Collecting terms of order ϵ in (1), (2), (6), (8) and (9) yields

$$\left.\begin{aligned} &\frac{\partial r_0}{\partial T}+\frac{\partial S_0}{\partial \Xi}+\frac{\partial s_1}{\partial \xi} = 0, \\ &r_0\frac{\partial \Theta_0}{\partial T}+S_0\left(\frac{\partial \Theta_1}{\partial \xi}+\frac{\partial \Theta_0}{\partial \Xi}\right)+s_1\frac{\partial \Theta_0}{\partial \xi} = \frac{\partial^2 \Theta_1}{\partial \xi^2}+2\frac{\partial^2 \Theta_0}{\partial \xi\,\partial \Xi}, \\ &r_0\frac{\partial \Psi_0}{\partial T}+S_0\left(\frac{\partial \Psi_1}{\partial \xi}+\frac{\partial \Psi_0}{\partial \Xi}\right)+s_1\frac{\partial \Psi_0}{\partial \xi} = L^{-1}\frac{\partial^2 \Psi_1}{\partial \xi^2}+2L^{-1}\frac{\partial^2 \Psi_0}{\partial \xi\,\partial \Xi}, \\ &r_1 = -r_0^2\Theta_1\gamma/(1-\gamma), \quad s_1 = r_1 u_0 + r_0[u_{-\infty}(\Xi)+u_1-\alpha_{0T}], \end{aligned}\right\} \tag{17}$$

where the expansion

$$u_{-\infty}(X) = u_{-\infty}(\Xi)+\epsilon\alpha_0 u_{-\infty X}(\Xi)+\epsilon^2[\alpha_1 u_{-\infty X}(\Xi)+\tfrac{1}{2}\alpha_0^2 u_{-\infty XX}(\Xi)]+\dots \tag{18}$$

has been required. The subscript X denotes a partial derivative, and a similar expansion will be needed later for $\mathbf{v}_{-\infty}(X)$. Avoiding a secular growth for s_1 in the range $\xi < 0$ necessitates $\partial S_0/\partial \Xi = 0$ for $\Xi < 0$, according to the first equation in (17); hence the condition $S_0^{(0)}(0) = 1$ gives $S_0^{(0)}(\Xi) = 1$ for $\Xi < 0$. Similar considerations of secular behaviour applied to the equations in (17) for $\xi > 0$ lead to the requirements that

$$\frac{\partial R_0}{\partial T}+\frac{\partial S_0}{\partial \Xi} = 0, \quad R_0\frac{\partial R_0}{\partial T}+S_0\frac{\partial R_0}{\partial \Xi} = 0, \quad R_0\frac{\partial B_0}{\partial T}+S_0\frac{\partial B_0}{\partial \Xi} = 0, \tag{19}$$

in the range $\Xi > 0$, where $R_0 = [1+C_0\gamma/(1-\gamma)]^{-1}$. The system (19) requires S_0/R_0 to be independent of Ξ, and admits general solutions that are functions of the single variable $\Xi - \int_0^T (S_0/R_0)\,dT$; application of the boundary conditions at $\Xi = 0$ stated above then gives, for $\Xi > 0$, $C_0^{(0)}(\Xi) = 1$, $C_0^{(1)}(\Xi) = 0$, $B_0^{(0)}(\Xi) = 0$, $B_0^{(1)}(\Xi) = 0$ and $S_0^{(0)}(\Xi) = 1$. The result that $S_0^{(0)}(\Xi) = 1$ everywhere corresponds to $m_0 = 1$, i.e. $u_{\mathrm{T}} = u_{\mathrm{L}}$ at the lowest order in ϵ.

With these results, integration of the first equation in (17) yields

$$s_1 = \begin{cases} (1-r_0)\,(S_0C_0)^{-1}(\partial C_0/\partial T) + S_1(\Xi) & (\xi < 0), \\ S_1(\Xi) & (\xi > 0). \end{cases} \tag{20}$$

Since Θ_1 and Ψ_1 cannot decrease to zero more slowly than Θ_0 and Ψ_0 as $\xi \to -\infty$ (cf. Van Dyke 1964), the second and third equations of (17) split into two independent parts for $\xi < 0$, giving

$$S_0\frac{\partial \Theta_1}{\partial \xi} = \frac{\partial^2 \Theta_1}{\partial \xi^2}, \quad S_0\frac{\partial \Psi_1}{\partial \xi} = L^{-1}\frac{\partial^2 \Psi_1}{\partial \xi^2}, \tag{21}$$

along with two other differential equations evident from (17). For $\xi > 0$, (16) and (19) show that (21) is again obtained from the second and third equations of (17), this time with no additional differential equations implied. The solutions to (21) satisfying the upstream and downstream boundary conditions are

$$\Theta_1 = \begin{cases} e^{\xi S_0}C_1(\Xi) & (\xi < 0), \\ C_1(\Xi) & (\xi > 0), \end{cases} \quad \Psi_1 = \begin{cases} e^{L\xi S_0}B_1(\Xi) & (\xi < 0), \\ B_1(\Xi) & (\xi > 0), \end{cases} \tag{22}$$

where the first two lines of (13) imply that $C_1^{(0)}(0) = B_1^{(0)}(0) = B_1^{(1)}(0) = 0$. The first of the additional differential equations obtained from (17) for $\xi < 0$ may be shown by use of (16) and (20) to reduce to $S_0\partial C_0/\partial \Xi = S_0S_1C_0 + \partial C_0/\partial T$, and the second may then be shown to require that $B_0 = 1 - C_0^L$ for $\Xi < 0$. Although $S_1(\Xi)$ must be known before this differential equation can be solved, the previously derived conditions $C_0^{(0)}(0) = S_0^{(0)}(0) = 1$ yield $\partial C_0^{(0)}/\partial \Xi|_{\Xi=0} = S_1^{(0)}(0)$ from the equation. A proper multiscale development of the fourth line of (13) in powers of ϵ then shows that $C_1^{(1)}(0) = 0$, which leads to $S_1^{(0)}(0) = 0$ by use of the third line of (13). Use of (1) and (11) in (20) for $\xi \to -\infty$ gives $S_1(\Xi) = u_{-\infty}(\Xi) + m_1 - \alpha_{0T}$, whence $S_1^{(0)}(0) = 0$ yields $\alpha_{0T}^{(0)} = u_{-\infty}(0) + m_1^{(0)}$, the time average of which shows that $m_1^{(0)} = 0$. Thus there is no correction to the turbulent flame speed at order ϵ, and we have $S_1^{(0)}(\Xi) = u_{-\infty}(\Xi) - u_{-\infty}(0)$ for $\Xi < 0$ and $\alpha_{0T}^{(0)} = u_{-\infty}(0)$, in agreement with previous results (Clavin & Williams 1979). Obtaining more information concerning dynamical properties of the flame front necessitates considering α_{1T} by proceeding to order ϵ^2.

Collecting terms of order ϵ^2 in (2) and (9) yields

$$\frac{\partial r_1}{\partial T} + \frac{\partial s_1}{\partial \Xi} + \frac{\partial s_2}{\partial \xi} + \hat{\nabla}.\{r_0[\mathbf{v}_1 + \mathbf{v}_{-\infty}(\Xi)]\} = 0, \tag{23}$$

$$\begin{aligned} r_0\frac{\partial \Psi_1}{\partial T} + r_1\frac{\partial \Psi_0}{\partial T} + S_0\left(\frac{\partial \Psi_2}{\partial \xi} + \frac{\partial \Psi_1}{\partial \Xi}\right) + s_1\left(\frac{\partial \Psi_1}{\partial \xi} + \frac{\partial \Psi_0}{\partial \Xi}\right) + (s_2 + L^{-1}\hat{\nabla}^2\alpha_0)\frac{\partial \Psi_0}{\partial \xi} \\ = -r_0[\mathbf{v}_1 + \mathbf{v}_{-\infty}(\Xi)].\hat{\nabla}\Psi_0 + L^{-1}\left[|\hat{\nabla}\alpha_0|^2\frac{\partial^2 \Psi_0}{\partial \xi^2} + \hat{\nabla}^2\Psi_0 + \frac{\partial^2 \Psi_2}{\partial \xi^2} + 2\frac{\partial^2 \Psi_1}{\partial \xi\,\partial \Xi} + \frac{\partial^2 \Psi_0}{\partial \Xi^2}\right], \end{aligned} \tag{24}$$

where $\hat{\nabla}$, an operator of order unity, denotes the transverse gradient involving differentiation with respect to Y and Z. The terms of order ϵ^2 in (6) provide an equation for

Θ_2 that can be obtained from (24) by replacing Ψ by Θ and L by unity. Since (23) and (24) involve $\mathbf{v}_1$, it becomes necessary to investigate (5) at order ϵ, viz

$$S_0\frac{\partial \mathbf{v}_1}{\partial \xi} = -\hat{\nabla} p_0 + (\hat{\nabla}\alpha_0)\frac{\partial p_0}{\partial \xi} + P\frac{\partial^2 \mathbf{v}_1}{\partial \xi^2} - P'\left[(\hat{\nabla}\alpha_0)\frac{\partial^2 u_0}{\partial \xi^2} - \hat{\nabla}\left(\frac{\partial u_0}{\partial \xi}\right)\right], \tag{25}$$

where u_0 and p_0 are obtained from the last three equations of (15) by use of (14), (16) and previously stated relationships for S_0, C_0 and B_0. It is found that

$$\left.\begin{aligned} u_0 &= \begin{cases} S_0[1+e^{\xi S_0}C_0(\Xi)\gamma/(1-\gamma)] & (\xi<0),\\ S_0/(1-\gamma) & (\xi>0),\end{cases}\\ p_0 &= \begin{cases} S_0^2(P+P'-1)\,e^{\xi S_0}C_0(\Xi)\gamma/(1-\gamma) & (\xi<0),\\ -S_0^2\gamma/(1-\gamma) & (\xi>0),\end{cases}\end{aligned}\right\} \tag{26}$$

there being a discontinuity of pressure across the reaction zone to balance the discontinuity of the normal viscous stress in (14). The solution to (25) for $\xi < 0$ that satisfies the upstream boundary condition is

$$\mathbf{v}_1 = (\hat{\nabla}\alpha_0)(m_0 - u_0) - \hat{\nabla}\int_{-\infty}^{\xi}[m_0 - u_0(\xi')]\,d\xi' + e^{\xi S_0/P}\,\mathbf{V}_1(\Xi), \tag{27}$$

where $\mathbf{V}_1(\Xi)$ is to be determined by considerations of secular behaviour at order ϵ^2. For $\xi > 0$, (25) with (26) shows that $\mathbf{v}_1$ must be independent of ξ to avoid a secular growth, and we choose to write the solution for $\xi > 0$ as $\mathbf{v}_1 = \mathbf{V}_1(\Xi) - (\hat{\nabla}\alpha_0)S_0/(1-\gamma)$, so that $\mathbf{V}_1(\Xi)$ will be continuous at $\Xi = 0$. The jump condition obtained from the expansion of (14) for the normal derivative of $\mathbf{v}_1$ then reduces to $\mathbf{V}_1^{(0)}(0) = 0$.

Having thus obtained a sufficient amount of information concerning $\mathbf{v}_1$, we may proceed to consider solving (23) and (24). Differentiation of the relation for $S_1(\Xi)$, obtained after (22), shows that $\partial S_1/\partial \Xi = u_{-\infty X}(\Xi)$, which equals $-\hat{\nabla}\cdot\mathbf{v}_{-\infty}(\Xi)$ by continuity in the undisturbed flow. Therefore for $\xi < 0$ there is no secular term in (23), and integration from $-\infty$ to ξ, with use of (20), (27) and the next-to-last equations in (15) and in (17), results in a long but explicit expression for s_2 that involves an undetermined function $S_2(\Xi)$ as a constant of integration with the property that s_2 approaches $S_2(\Xi)$ as $\xi \to -\infty$. Evaluation of this expression at $\Xi = 0$, with the aid of (16), (22), the fourth equation in (15) and previously derived conditions at $\Xi = 0$ for the functions that vary only on the longer distance scale, gives

$$s_2|_{\Xi=0} = S_2(0) + \left[\hat{\nabla}^2\alpha_0 - \frac{u_{-\infty X}(0)}{S_0} - \frac{1}{S_0^2}\frac{\partial S_1}{\partial T}\right]\ln\left(1+\frac{\gamma}{1-\gamma}e^{\xi S_0}\right), \tag{28}$$

valid to order β^{-1}. The same kind of splitting that led to (21) applies to (24) and gives, for $\Xi = 0$ and $\xi < 0$,

$$S_0\frac{\partial \Psi_2}{\partial \xi} - L^{-1}\frac{\partial^2 \Psi_2}{\partial \xi^2} = \left[\hat{\nabla}^2\alpha_0 - \frac{u_{-\infty X}(0)}{S_0} - \frac{1}{S_0^2}\frac{\partial S_1}{\partial T}\right]LS_0 e^{L\xi S_0}\ln\left(1+\frac{\gamma}{1-\gamma}e^{\xi S_0}\right), \tag{29}$$

as well as $S_0\partial B_1/\partial\Xi = LS_0(S_0|\hat{\nabla}\alpha_0|^2 - S_2) - S_0\hat{\nabla}^2\alpha_0 - LS_1^2 - L^{-1}\partial^2{}_0/\partial\Xi^2 B$. Replacing Ψ by Θ, B by C, and L by unity reduces these equations to those obtained for temperature from (6). Differentiation of earlier results for B_0 and C_0 shows that

$$\partial^2 B_0/\partial \Xi^2 = -L[u_{-\infty X} + S_1^2 + S_0^{-1}\partial S_1/\partial T] - L(L-1)(\partial C_0/\partial \Xi)^2$$

and $\partial^2 C_0/\partial\Xi^2 = u_{-\infty X}+S_1^2+S_0^{-1}\partial S_1/\partial T$, which may be used to simplify the equations split on the longer scale, thereby providing expressions for $\partial B_1/\partial\Xi$ and $\partial C_1/\partial\Xi$, evaluated at $\Xi = 0$. Integration of (29) from $\xi = -\infty$ to $\xi = 0$ provides a relationship between values and slopes at $\xi = 0-$ for Ψ_2; a corresponding relationship for Θ_2 is obtained similarly. These relationships may be used in the expansion to order ϵ^2 of the last jump condition in (13) to obtain σ_2. This entails employing $\Psi_1 = \Theta_1 = 0$ for $\xi > 0$ to the first two orders in β^{-1}, a result obtained from (24) and from the corresponding equation for Θ_2 by a procedure paralleling that involving (19). The formula for σ_2 is found to be

$$\sigma_2 = l[u_{-\infty X}(0)-\hat{\nabla}^2\alpha_0]\frac{1-\gamma}{\gamma}\int_0^{\gamma/(1-\gamma)} x^{-1}\ln(1+x)\,dx, \tag{30}$$

after the previously derived results that $S_0^{(0)}(0) = 1$ and $S_0^{(0)}(0) = 0$ are reintroduced. Equation (30) defines a departure of the non-dimensional flame temperature from its adiabatic value by an amount of order ϵ^2. This result is equivalent to (B 20), in view of (B 11).

The third line of the jump condition (13), extended to order ϵ^2, gives

$$S_2^{(0)}(0) = [u_{-\infty X}(0)-\hat{\nabla}^2\alpha_0^{(0)}]\frac{1}{\gamma}\ln\left(\frac{1}{1-\gamma}\right)+\tfrac{1}{2}\sigma_2+\tfrac{1}{2}|\hat{\nabla}\alpha_0^{(0)}|^2. \tag{31}$$

Use of (1) and of the formula for s_2 in the limit $\xi\to-\infty$ yields

$$S_2^{(0)}(0) = m_2^{(0)}-\alpha_{1T}^{(0)}-\hat{\nabla}.[\alpha_0^{(0)}\mathbf{v}_{-\infty}(0)],$$

where (2) and (18) have been employed. Substitution of (30) and (31) into this formula provides us with the equation of evolution of the flame front to order ϵ^2, viz

$$\begin{aligned}\alpha_{1T} &= m_2-\tfrac{1}{2}|\hat{\nabla}\alpha_0|^2-\hat{\nabla}.[\alpha_0\mathbf{v}_{-\infty}(0)]\\ &\quad -[u_{-\infty X}(0)-\hat{\nabla}^2\alpha_0]\left[\frac{1}{\gamma}\ln\left(\frac{1}{1-\gamma}\right)+\tfrac{1}{2}l\left(\frac{1-\gamma}{\gamma}\right)\int_0^{\gamma/(1-\gamma)} x^{-1}\ln(1+x)\,dx\right],\end{aligned} \tag{32}$$

where the superscript (0) has been left out for brevity. Equation (32) describes the departure of the motion of the fluctuating flame from the fluctuating motion of the gas in the fresh mixture. The significance of (32) will be discussed in § 5. Equations (B 10) and (B 11) show that (32) is the same as (B 21).

In view of transverse homogeneity, the time average of (32) yields $m_2 = \tfrac{1}{2}\overline{|\hat{\nabla}\alpha_0|^2}$, which may be written as $\epsilon^2 m_2 = \tfrac{1}{2}\overline{|\nabla\alpha_0|^2}$. Since in the expansion

$$m = m_0+\epsilon m_1+\epsilon^2 m_2+\dots$$

it was found previously that $m_0 = 1$ and $m_1 = 0$, this result shows that the turbulent flame speed exceeds the laminar flame speed by the fractional amount $\tfrac{1}{2}\overline{|\nabla\alpha_0|^2}$, of order ϵ^2. This flame-speed correction, also found in (B 22), is merely the first non-vanishing correction term in the low-intensity expansion of the purely geometrical effect given by (A 7). Therefore local modifications to structures of laminar flamelets do not influence the turbulent flame speed at order ϵ^2; only the effect of area increase associated with flame wrinkling appears. To obtain the first non-geometrical corrections to m, the analysis must be carried to order ϵ^4. This higher-order analysis has been completed only for $\gamma = 0$, only for averaged quantities and under the assumption that the stochastic process is stationary as well as homogeneous in transverse directions. Some of the details are given in appendix C. At order ϵ^3 only a further geometrical effect appears, which can be obtained by expansion of (A 7). At order ϵ^4, additional structural effects enter. The results through order ϵ^4 are consistent with the summary formula

$$m = \overline{[1+|\nabla\alpha|^2]^{\frac{1}{2}}} - l(1+\tfrac{1}{8}l)\,\overline{[\epsilon u_{-\infty X}(0) - \nabla^2\alpha]^2}, \qquad (33)$$

which applies to the purely diffusive model obtained in the limit in which γ vanishes.

In connection with possible future applications of the multiscale method, it is of interest to remark that the last equation of (13) provides the expansion in ϵ of the local instantaneous temperature σ of the flame independently of the third line of (13), which can be used afterward to obtain a resummed form of an equation for evolution of the flame front. Pursuit of this line of investigation toward obtaining a nonlinear evolution equation may shed more light on phenomena of local quenching of the flame by stretch, discussed in the following section. The quantity S_0 and the time derivative of S_1 were retained in the developments leading to (28) and (29) to provide a starting point for this further analysis.

5. Motion of the flame front

The result $\alpha_{0\mathrm{T}} = u_{-\infty}(0)$ describes a flame front that moves with the streamwise Eulerian displacement of the turbulence. The correction to this provided by (32) includes a number of new physical phenomena that cause the fluctuating motion of the flame to depart from that of the gas. The first three terms on the right-hand side of (32) describe approximations to the kinematic effects summarized by (12) and discussed in appendix A. The identity $\hat{\nabla}\cdot[\alpha_0 \mathbf{v}_{-\infty}(0)] = \mathbf{v}_{-\infty}(0)\cdot\hat{\nabla}\alpha_0 - \alpha_0 u_{-\infty X}(0)$ aids in revealing this correspondence. The term $\epsilon\alpha_0 u_{-\infty X}(0)$ is an approximation to $u|_{x=\alpha} - u|_{x=0}$ and therefore arises from expansion of the first term in the expression (12) for α_t; it accounts for the fact that the fluctuating velocity at the flame is not exactly that at $x = 0$. The term $\epsilon \mathbf{v}_{-\infty}(0)\cdot\hat{\nabla}\alpha_0$ is an approximation to the second term in (12), accounting for transverse convection into a tilted flame element. The first two terms in (32) represent an expansion of the last two in (12), describing influences of area increase. The remaining terms in (32) represent influences of turbulent modifications to the structure of the wrinkled laminar flame and are not contained in (12).

Consider first the limit $\gamma \to 0$ in which effects of gas expansion through the laminar flame are absent. In this limit the additional terms in (32) reduce to

$$-(1+\tfrac{1}{2}l)\,[u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0].$$

Here $u_{-\infty X} = -\hat{\nabla}\cdot\mathbf{v}_{-\infty}$ makes it clear that the first term in the brackets describes an influence of flame stretch of the type analysed by Eckhaus (1961), Klimov (1963) and Sivashinsky (1976). This effect has also been investigated by Joulin (1979) within the context of the expansion $L = 1+\beta^{-1}l$ for steady flames in stagnation-point flows. It is found (Sivashinsky 1976) that, in the appropriate range of values of l, sufficiently strong stretch can produce extinction even under adiabatic conditions and that for weak stretch the stretch effects are proportional to $1+\frac{1}{2}l$. The term $-(1+\frac{1}{2}l)\,u_{-\infty X}(0)$ in (32) is an unsteady, multidimensional version of the weak-stretch effect. This acts on $\alpha_{1\mathrm{T}}$ in an additive manner with $(1+\frac{1}{2}l)\,\hat{\nabla}^2\alpha_0$, the diffusional effect of transverse curvature on flame motion, sought to explain the experimental observations discussed in § 1. The occurrence of these phenomena in (32) suggests that the present analysis may be a good starting point for further investigation of stretch and curvature effects on flame-front motion.

The factor $1+\frac{1}{2}l$ is the well-known bifurcation parameter that arises in analyses of diffusive-thermal stability of laminar flames (Joulin & Clavin 1979). Planar flames are diffusively unstable if this parameter is negative, as may readily be inferred from a linearized simplification to (32). Specifically, writing $\alpha_{1T} = (1+\frac{1}{2}l)\hat{\nabla}^2\alpha_0$ and then disregarding ordering, it is seen that, if $1+\frac{1}{2}l$ is negative, then a positive curvature implies a negative α_t and therefore a tendency for the curvature to increase further with time. Thus, it is evident that (32) effectively retains the property of instability for negative values of the bifurcation parameter and should not be presumed to describe turbulent flames under conditions of instability.

Analyses of diffusive-thermal instability have been completed only for the limit $\gamma \to 0$. Equation (32) implies that for $\gamma \neq 0$ effects of gas expansion will modify the bifurcation parameter; $1+\frac{1}{2}l$ is replaced by the quantity in the last square brackets of (32). Since $\gamma^{-1}\ln(1-\gamma)^{-1}$ is an increasing function of γ that approaches infinity as γ approaches unity, it is seen that for $l = 0$ the characteristic relaxation time decreases as γ increases; gas expansion quickens the recovery of the stable flame from a condition of distortion. A typical value of γ for a real flame is 0·85, and the corresponding factor for enhancement of the relaxation rate is 2·2 for $l = 0$. If γ becomes too close to unity, then the assumption that flame-front motion occurs on a time scale such that changes in ϵt are of order unity will no longer be correct, and a revision of the scaling adopted herein will be needed.

There is also a modification of the critical value of l below which instability occurs. From (32), (B 10) and (B 11) it is seen that this critical value is given by

$$l^* = -\frac{2}{1-\gamma}\ln\left(\frac{1}{1-\gamma}\right)\left[\int_0^{\gamma/(1-\gamma)} x^{-1}\ln(1+x)\,dx\right]^{-1}$$

$$= \left\{\begin{array}{l} -2(1+\frac{3}{4}\gamma+\ldots) \quad (\gamma\to 0), \\ -(48/\pi^2)\ln 2 = -3{\cdot}37 \quad (\gamma = \frac{1}{2}), \\ -4\left[(1-\gamma)\ln\left(\dfrac{1}{1-\gamma}\right)\right]^{-1}+\ldots \quad (\gamma \to 1). \end{array}\right\} \qquad (34)$$

This result shows that l^* decreases from -2 as γ increases, approaching $-\infty$ as γ approaches unity. Thus, effects of gas expansion can significantly decrease the critical value of the Lewis number below which diffusive-thermal instability occurs. This effect arises from the fact that the factor multiplying l in (32) is a decreasing function of γ. For realistic values of γ, the decrease of l^* below -2 is substantial; for example for $\gamma > 0{\cdot}8$, $l^* < -10$, which causes $l > l^*$ for most combustible mixtures, possibly excluding those having hydrogen as fuel. Therefore, existing studies of cellular flames deserve re-evaluation with respect to possible influences of gas expansion; Pelcé & Clavin (1982) give a relevant analysis. The growth rates of instabilities in diffusively unstable regimes are reduced by effects of gas expansion.

A composite equation for evolution of a flame front in a turbulent flow may be written by resumming the results that have been derived, including the findings of (12) and (32). The result may be expressed in the notation of (12) as

$$\alpha_t = u|_{x=\alpha} - \mathbf{v}|_{x=\alpha}\cdot\nabla\alpha + m - [1+|\nabla\alpha|^2]^{\frac{1}{2}}\left[1-(u_x|_{x=\alpha}-\nabla^2\alpha)\left(1-\frac{l}{l^*}\right)\frac{1}{\gamma}\ln\left(\frac{1}{1-\gamma}\right)\right], \quad (35)$$

where l^* is given by (34), and m is given by (12). For $\gamma = 0$, (33) might be used instead of (12) for m in (35), with $\epsilon u_{-\infty X}(0)$ replaced by $u_x|_{x=\alpha}$, although there would then be an inconsistency in that the difference between (33) and (12) is a term of order ϵ^4, while elsewhere in (35) only terms up to order ϵ^2 have been retained. By the same re-identification, (30) for the flame temperature may be written as

$$\frac{T|_{\xi=0} - T_f}{T_f - T_0} = (L-1)\,(u_x|_{x=\alpha} - \nabla^2\alpha)\left(\frac{1-\gamma}{\gamma}\right)\int_0^{\gamma/(1-\gamma)} x^{-1}\ln(1+x)\,dx. \qquad (36)$$

In these formulas, lengths, times and velocities are non-dimensionalized by the laminar-flame values, and u and $\mathbf{v}$ are functionals of α.

Equation (35) may be proposed as a nonlinear equation describing the motion of the flame front in a non-uniform velocity field. It therefore deserves comparison with the earlier equation for flame-front evolution proposed by Sivashinsky (1977*b*). Since the earlier work contained neither the effects of non-uniform velocities nor the effects of gas expansion, the dependences on u, $\mathbf{v}$ and γ in (35) are not present in earlier work. Sivashinsky (1977*b*) included a term representing an approximation to the effect of hydrodynamic instability, but no such term appears in (35) since these effects have been ignored herein, as previously discussed.† It is generally recognized that the earlier approximation $1+\frac{1}{2}|\nabla\alpha|^2$ to the geometrical term $[1+|\nabla\alpha|^2]^{\frac{1}{2}}$ becomes inaccurate as $|\nabla\alpha|^2$ increases. Finally, a term $-\nabla^4\alpha$ should appear on the right-hand side of (35), according to Sivashinsky (1977*b*). In stability analyses of laminar flames this term is responsible for the occurrence of stability at short wavelengths even when $l < l^*$. The term would have arisen in the present analysis if the expansion had been carried fully to order ϵ^4. Therefore if (35) is to be employed as a general equation, including descriptions of flame evolution under diffusively unstable conditions, then $-\nabla^4\alpha$ must be added to the right-hand side. The modification of the coefficient of $-\nabla^4\alpha$ by gas expansion remains to be calculated. In stable situations this term is less important, but $\nabla^2\alpha$ in (35) remains significant at low frequencies, as will be seen when the predictions of (35) concerning figure 1 are discussed.

It is anticipated that the assumption of low turbulence intensity is not essential to the major results obtained here, and can be removed by future work. Increased turbulent intensity is expected to produce purely kinematic nonlinearities of the type treated in appendix A and summarized in (12). On the other hand, the assumption of large-scale turbulence is critical, since the local structure of the wrinkled flame is strongly influenced by the ratio of the flame thickness to the turbulence scale. A promising avenue for future work is the inclusion of the nonlinearity responsible for the production of local quenching by flame stretch. An approach to this problem has been suggested at the end of § 4.

6. Flame temperature and turbulent flame speed

Phenomena that cause the local, instantaneous flame temperature to differ from the adiabatic flame temperature T_f in the absence of heat losses (e.g. by radiation) to the surroundings appear in (36). Specifically, flame stretch u_x and flame curvature $\nabla^2\alpha$ introduce departures of the flame temperature from T_f if $L \neq 1$. Positive stretch

† Pelcé & Clavin (1982) use the local relationship (35) to address the hydrodynamic problem, taking into account the modifications of u and $\mathbf{v}$ associated with the influence of combustion on the upstream flow when $\gamma \neq 0$.

(i.e. negative u_x) and positive curvature lead to an increased flame temperature if $L < 1$. This is understandable, since each of these phenomena increases upstream gradients and thereby enhances reactant transport (i.e. heat release) to a greater extent than conductive energy transport if $L < 1$. Thus, (36) predicts a variation of flame temperature along the flame sheet for a wrinkled flame or for a planar flame in a strain field.

Gas expansion reduces the sensitivity of flame temperature to stretch and curvature. This is seen from the fact that the γ-dependence on the right-hand side of (36) is represented by a decreasing function of γ that goes from unity at $\gamma = 0$ to zero at $\gamma = 1$, as summarized in (B 11). A reduction in sensitivity by more than a factor of 2 may be expected for reasonable values of γ. It seems physically reasonable that, since gas expansion tends to lessen gradients, it may diminish the influence of stretch and curvature on the flame temperature.

Associated with an increase in flame temperature is an increase in flame speed. Therefore the modifications to flame temperature in (36), produced diffusively as consequences of stretch and curvature, will be reflected in local modifications to the propagation velocity of the wrinkled laminar flame. The average of these flame-speed modifications provides an additive contribution to the turbulent flame speed. This contribution is given in (33) for $\gamma = 0$, under conditions of weak stretch and weak curvature. The mean square of the sum of the stretch and curvature terms appears in (33) because the averages of the linear terms vanish. It is for this reason that the analysis had to be carried to order ϵ^4 to obtain (33). Since gas expansion reduces the effects of stretch and curvature on flame temperature, it seems likely that for $\gamma \neq 0$ the magnitude of the coefficient of $\overline{(u_x|_{x=\alpha} - \nabla^2\alpha)^2}$ in (33) will decrease as γ increases, approaching zero as γ approaches unity. Markstein (1964) has properly interpreted $\nabla^2\alpha - u_x|_{x=\alpha}$ as the curvature of the flame with respect to that of the flow.

From (33) it is seen that, for $l > 0$ (i.e. for $L > 1$), these diffusively related effects of stretch and curvature provide a decrease of the turbulent flame speed below the value obtained by considering only the effect of the area increase of the wrinkled flame. On the other hand, for $l < 0$, a condition that is encountered more often in practice, stretch and curvature contribute an increase to the turbulent flame speed. Since (33) is valid only for the stable regime, $l > -2$ if $\gamma = 0$, conclusions concerning influences of stretch and curvature in diffusively unstable situations cannot be drawn from (33). Sufficiently accurate measurements of turbulent flame speeds, to test the predictions of (33) with regard to effects of stretch and curvature, are not yet available.

7. Influence of the flame on the turbulence

One result of the present analysis is a prediction of the change in the velocity field that occurs upon passage of the turbulence through the flame. This prediction is obtained from the results of the first-order analysis, given in the paragraph containing (27), or between (B 5) and (B 6). Specifically, it is found that, through the wrinkled flamelet located at $\Xi = 0$, $u_1 = 0$ and $\mathbf{v}_1 = (1-u_0)\hat{\nabla}\alpha_0$. Thus, from (11), near the turbulent flame ($X = 0$),

$$U = u_0(\xi) + \epsilon u_{-\infty}(0, Y, Z, T) + O(\epsilon^2), \tag{37}$$

$$\mathbf{v} = \epsilon \mathbf{v}_{-\infty}(0, Y, Z, T) + [\nabla\alpha_0(Y, Z, T)][1 - u_0(\xi)] + O(\epsilon^2). \tag{38}$$

Ahead of the flame $u_0 = 1$, and behind $u_0 = (1-\gamma)^{-1}$. Thus, behind the flame the fluctuating components of the streamwise and transverse velocities are

$$u_L \epsilon u_{-\infty}(0, Y, Z, T)$$

and $u_L \epsilon \mathbf{v}_{-\infty}(0, Y, Z, T) - u_L \nabla\alpha_0(Y, Z, T)\gamma/(1-\gamma)$, respectively, corresponding simply to flow-velocity deflections by the tilted elements of the flame sheet.

These results show that the flame does not influence the longitudinal component of the fluctuating velocity; it merely produces a uniform increase in velocity, from u_L to $u_L(1-\gamma)$, through gas expansion. On the other hand, the transverse velocity is modified as a consequence of passage of fluid elements through tilted elements of the flame sheet, if $\gamma \neq 0$. As γ approaches unity, this modification becomes large. The change in the transverse velocity is seen to exhibit no explicit dependence on the upstream transverse velocity but instead to be controlled by the transverse gradient of the streamwise Eulerian displacement upstream, a quantity dependent on the fluctuating longitudinal velocity. Thus, effects of gas expansion can alter turbulence properties downstream significantly.

Let $\hat{u}$ and $\hat{\mathbf{v}}$ denote the fluctuating longitudinal and transverse velocities just upstream from the turbulent flame. Then the turbulent kinetic energy per unit mass upstream is $q_{-\infty} = \frac{1}{2}[\overline{\hat{u}^2} + \overline{|\hat{\mathbf{v}}|^2}]$, and that just downstream is

$$q_\infty = \tfrac{1}{2}\{\overline{\hat{u}^2} + \overline{|\hat{\mathbf{v}} - (\nabla\alpha_0)\, u_L \gamma/(1-\gamma)|^2}\}. \tag{39}$$

When the gas expansion becomes large, γ approaches unity and (39) becomes $q_\infty \simeq \frac{1}{2}\overline{(\nabla\alpha_0)^2}\, u_L^2 \gamma^2/(1-\gamma)^2$, which is large. Thus, for γ near unity the flame produces a substantial increase in the turbulent kinetic energy per unit mass, this increase appearing entirely in transverse components of the fluctuations. Of course, in the opposite limit, $\gamma \to 0$, there is no change in q across the flame. It might appear from (39) that there is a possibility of a decrease in q if γ is not too large. However, this seems unlikely since, in the cross term, $\overline{\hat{\mathbf{v}}\,.\,\nabla\alpha_0} = \overline{\hat{u}_X \alpha_0}$ by virtue of continuity and homogeneity in transverse directions, whence invocation of an hypothesis of stationarity and a Taylor hypothesis gives $\overline{\hat{\mathbf{v}}\,.\,\nabla\alpha_0} = -\overline{\hat{u}^2}/u_L$. Thus, the cross-correlation that occurs in (39) appears to provide a further increase in q, not a tendency toward cancellation. If the preceding approximations and the first term in the expansion of (33) for small ϵ are employed, then

$$q_\infty - q_{-\infty} = (u_T - u_L)\, u_L \gamma^2/(1-\gamma)^2 + \overline{\hat{u}^2}\gamma/(1-\gamma) \tag{40}$$

is obtained as an approximate expression for the increase in q produced by the turbulent flame. Equation (40) may be useful for comparison with experiment in that it contains quantities most readily measurable; all of the properties of u_T discussed by Clavin & Williams (1979) are applicable in (40), to the order to which this expression holds true, and therefore under suitable assumptions $(u_T - u_L)\, u_L = \overline{\hat{u}^2}$ in (40).

The turbulent kinetic energy per unit volume is ρq. This quantity may decrease when the turbulence passes through the flame because the density decreases. If the approximations of (40) are employed, then it is found that

$$(\rho q)_\infty - (\rho q)_{-\infty} = [(u_T - u_L)\, u_L \gamma^2/(1-\gamma) - \tfrac{1}{2}\gamma(\overline{|\hat{\mathbf{v}}|^2} - \overline{\hat{u}^2})]\,\rho_0, \tag{41}$$

which is negative for small values of γ. However, as γ approaches unity this quantity too becomes large and positive.

Unlike these kinetic energies, the intensity of the turbulence relative to the mean velocity does not diverge as γ approaches unity. The longitudinal and transverse components of the relative intensity upstream may be defined as $(\overline{\hat{u}^2})^{\frac{1}{2}}/\hat{U}$ and $(\overline{|\hat{\mathbf{v}}|^2})^{\frac{1}{2}}/\hat{U}$, respectively, where $\hat{U}$ upstream is u_L in the first approximation. Since the corresponding value of $\hat{U}$ downstream is $u_L/(1-\gamma)$, the relative intensity for the streamwise component of velocity decreases in passage through the flame, its downstream value being $(1-\gamma)\,(\overline{\hat{u}^2})^{\frac{1}{2}}/u_L$. This aspect of the prediction is consistent with the concept of reduction in the relative turbulence intensity through dilatation. Similarly, with the approximation of (40), the relative intensity in transverse fluctuations downstream is $u_L^{-1}[\overline{|\hat{\mathbf{v}}|^2}(1-\gamma)^2+2\overline{\hat{u}^2}\gamma(1-\gamma)+2(u_T-u_L)\,u_L\,\gamma^2]^{\frac{1}{2}}$, which as γ increases typically first decreases, then reaches a minimum value (at $\gamma = \overline{\hat{u}^2}/[2u_L(u_T-u_L)]$ if the turbulence is initially isotropic) and afterward increases, approaching, as γ approaches unity, the value $[2(u_T-u_L)/u_L]^{\frac{1}{2}}$, equal to its initial value for conditions under which $(u_T-u_L)\,u_L = \overline{\hat{u}^2}$. Since these relative intensities remain bounded, the influence of the flame on the turbulence does not introduce any restrictions on the range of γ for which the analysis may be applied without contradicting initial premises.

Among the consequences of these results is the conclusion that initially isotropic turbulence will not be isotropic after passage through the flame. Transverse fluctuations are enhanced with respect to longitudinal fluctuations. For example, under the full set of approximations introduced above, the ratio of a transverse component of intensity to the longitudinal component is

$$\left[1+\frac{\gamma}{(1-\gamma)^2}\right]^{\frac{1}{2}}$$

downstream from the flame. This ratio increases with γ and becomes infinite as γ approaches unity. The increase in the turbulent kinetic energy may be viewed as a kind of flame-generated turbulence. However, such generation occurs entirely in transverse fluctuations, according to the present results.

8. Power spectrum of the flame velocity

To investigate the results shown in figure 1, it is helpful to take a Fourier transform of α. Let ω and $\mathbf{k}$ be the non-dimensional frequency and transverse wavenumber vector, respectively, and let the tilde $\sim$ denote the Fourier transform. To lowest order in ϵ, (35) is $\alpha_t = u|_{x=0}$. The remaining terms on the right-hand side are $O(\epsilon^2)$. For small ϵ, all of these terms will be dominated by $u|_{x=0}$, except the transverse-curvature term $\nabla^2\alpha$, which will be seen to be relatively important at low frequencies. Note, for example, that for sufficiently small intensities the nonlinear terms tend to become negligible in comparison with $\nabla^2\alpha$. Therefore only $u|_{x=0}$ and the $\nabla^2\alpha$ term will be retained on the right-hand side. With the abbreviation

$$K = \left(1-\frac{l}{l^*}\right)\frac{1}{\gamma}\ln\left(\frac{1}{1-\gamma}\right),$$

the Fourier transform of (35) in time and in transverse spatial co-ordinates then becomes

$$(i\omega+Kk^2)\,\tilde{\alpha}(\mathbf{k},\omega) = \tilde{u}(\mathbf{k},\omega). \tag{42}$$

The non-dimensional space–time power spectrum of $u|_{x=0}$ may be defined as $s_{uu}(k,\omega) = \langle\tilde{u}(\mathbf{k},\omega)\,\tilde{u}^*(\mathbf{k},\omega)\rangle\,d\mathbf{k}\,d\omega$, where $\langle\;\rangle$ denotes an ensemble average and $*$ the complex conjugate. Let $s_{\dot{\alpha}\dot{\alpha}}(k,\omega)$ represent the corresponding spectrum of α_t. Then (42) implies that

$$s_{\dot{\alpha}\dot{\alpha}}(k,\omega) = s_{uu}(k,\omega)\,\omega^2/(\omega^2+K^2k^4). \tag{43}$$

The non-dimensional power spectrum in frequency alone, $s_{uu}(\omega)$, is recovered from $s_{uu}(k,\omega)$ by $s_{uu}(\omega) = \int s_{uu}(k,\omega)\,2\pi k\,dk$, the integral being carried over the entire wavenumber range of the spectrum. There will be a lower limit k_c, for the absolute value of the non-dimensional transverse wavenumber below which the spectrum must vanish because of confinement of the flow by the walls of the apparatus. The value of k_c will be of the order of the ratio of the laminar flame thickness d to the width d_c of the duct in which the flame is studied. Use of (43) then provides the formula

$$s_{\dot{\alpha}\dot{\alpha}}(\omega) = \int_{k_c}^{\infty} \frac{2\pi\omega^2 s_{uu}(k,\omega)}{\omega^2+K^2k^4}\,k\,dk \tag{44}$$

for the non-dimensional spectrum of the flamelet velocity.

From (44) it is seen that, if $\omega \gg Kk^2$ for all values of k over which $s_{uu}(k,\omega)$ differs appreciably from zero, then K^2k^4 may be neglected in the denominator, and

$$s_{\dot{\alpha}\dot{\alpha}}(\omega) \approx s_{uu}(\omega).$$

This corresponds to the result of Clavin & Williams (1979) and agrees with figure 1 at high frequencies. If the stated condition is not fulfilled, then K^2k^4 must be retained in the denominator, and the spectra of flame and gas velocities will differ. Since K^2k^4 came from the $\nabla^2\alpha$ term describing diffusive relaxation, it is clear that this phenomenon cannot be neglected at low frequencies.

Even without a detailed knowledge of $s_{uu}(k,\omega)$, (44) provides the behaviour of $s_{\dot{\alpha}\dot{\alpha}}(\omega)$ at low frequencies. Since $(1+k^4K^2/\omega^2)^{-1}$ is a Lorentzian function of k^2 with width ω/K, if ω is sufficiently small and if $s_{uu}(k,\omega)$ may be treated as approaching a finite, non-zero value as k approaches zero, then (44) yields

$$s_{\dot{\alpha}\dot{\alpha}}(\omega) \simeq \frac{s_{uu}(0,\omega)\,\pi\omega}{K}\left[\tfrac{1}{2}\pi - \arctan\frac{Kk_c^2}{\omega}\right]. \tag{45}$$

This result relies on the approximation that the Lorentzian width is smaller than the characteristic wavenumber over which $s_{uu}(k,\omega)$ varies. Since the latter is expected to be determined by the integral scale L_1, (45) needs $(\omega/K)^{\frac{1}{2}} < d/L_1$. With d related to the transit time t_L through the laminar flame by $d = u_L t_L$, and with L_1 linked to the characteristic turbulence time t_T by the Taylor-type hypothesis $L_1 = u_L t_T$, it is seen that in terms of the dimensional frequency ν this restriction becomes $\nu < Kt_L/t_T^2$. At these frequencies the variation of $s_{uu}(0,\omega)$ with ω is not likely to be significant, since the range of ν over which the major variation occurs is expected to be of order $1/t_T$. Hence, under the assumption that for practical purposes $s_{uu}(0,\omega)$ approaches a finite, non-zero limit as ω approaches zero, the function $s_{uu}(0,\omega)$ should be replaced by the constant $s_{uu}(0,0)$ in (45), within the accuracy of this equation.

Two spectral subregimes may be identified from (45). At extremely low frequencies such that $\nu \ll KD_{th}/d_c^2$, the argument Kk_c^2/ω is large, and (45) becomes approximately, in dimensional form,

$$S_{\dot{\alpha}\dot{\alpha}}(\nu) \simeq S_{uu}(0,0)\,\pi d_c^2\nu^2/K^2D_{th}^2. \tag{46}$$

The quadratic dependence on ν at small values of ν predicted by (46) is consistent with the shape of the spectrum shown in figure 1. At larger values of ν, such that $KD_{\text{th}}/d_c^2 \ll \nu < Kt_L/t_T^2$, in (45) Kk_c^2/ω is small, and the dimensional result

$$S_{\dot{\alpha}\dot{\alpha}}(\nu) \simeq S_{uu}(0,0)\,[\pi^2\nu/2K\,D_{\text{th}} - \pi/d_c^2] \tag{47}$$

is obtained. The linear dependence in (47) agrees with the shape of the spectrum of flamelet velocity shown in figure 1 for ν between roughly 0·5 and 2 Hz. Thus, it seems clear that (44) describes properly the observed spectrum $S_{\dot{\alpha}\dot{\alpha}}(\nu)$, at least in a qualitative manner. Detailed quantitative comparisons would require knowledge of $S_{uu}(0,0)$. However, $S_{uu}(0,0)$ may be eliminated between (46) and (47) and a comparison sought on the basis of a relationship among curvature, slope and intercept of the spectrum $S_{\dot{\alpha}\dot{\alpha}}(\nu)$. The comparisons performed thus far have been successful (Boyer *et al.* 1981).

From (47) it is seen that the slope of the linear portion of the spectrum of flamelet velocity fluctuations depends on the bifurcation parameter, related to K, and the intercept depends on the cut-off wavelength d_c. Quantitative information concerning both of these quantities may therefore be obtained from power spectra $S_{\dot{\alpha}\dot{\alpha}}(\nu)$. Existing data of Sabathier (1980) are useful for this purpose. Further measurements of the power spectra for different fuels would be of interest for obtaining additional information of this type. It would be particularly interesting to investigate the variation of the bifurcation parameter with gas composition in this way. For the existing experiments complications arise in accurate comparisons because the nonlinear terms of (35) are not entirely negligible.

9. Concluding remarks

Inclusion of fractional changes in density of order unity in the present analysis adds a considerable amount of confidence to the application of the results to real turbulent flames. However, it should be emphasized that taking $\gamma \neq 0$ provided only quantitative modifications to the results concerning flamelet temperature and flamelet motion; no qualitatively new effects on these properties arose. This observation lends support to the relevance of analyses concerned with the limit $\gamma \to 0$. Often the restriction $\gamma \to 0$ is helpful for simplifying studies of complex flows, and the present results suggest that useful information can be obtained on the basis of this approximation.

The quantitative influences of $\gamma \neq 0$ may be important in various respects. Particularly notable are the quickening of the flamelet relaxation with increasing γ for $L = 1$ and the reduction in the critical Lewis number below which diffusive-thermal instability occurs, as given by (34). The latter effect especially is associated strongly with the weakening of the influences of strain rate and flamelet curvature on the flamelet temperature as γ increases. Quantitative effects of $\gamma \neq 0$ may also be significant in extracting information from measurements of power spectra of flamelet velocities.

Consideration of $\gamma \neq 0$ enables predictions to be made of influences of the flame on the turbulence. It has been seen that the principal new phenomenon obtained is the enhancement of transverse-velocity fluctuations through effects of gas expansion in passage through the flame. Only relationships between quantities just upstream ($-\infty$) and just downstream ($+\infty$) of the turbulent flame brush have been calculated here. The hydrodynamical effects that occur in the far field both downstream and upstream if $\gamma \neq 0$ were not analysed. There are a number of reasons to believe that in most situations these effects will not produce strong modifications, but nevertheless it would be of interest to consider them further on the basis of the present approach.

The analysis for small intensity and large scale is atypical in that proceeding to higher orders introduces a variety of new physical phenomena. It is necessary to include orders ϵ^0, ϵ^1 and ϵ^2 to obtain diffusive-thermal effects, and it is necessary to proceed to order ϵ^4 in the mean to calculate a nongeometrical correction to the turbulent flame speed. It is fortunate that the associated simplifications enable the expansion to be developed to such high orders. The most useful predictions, e.g. (35) and (36), are obtained after resumming the expansion. Improvement over the method of re-summing as applied to the solution may be offered by an approach based on the direct development of an expansion of the operator without obtaining the solution. More thorough use of existing multiple-scale methods, as well as further advances in such methods, would aid in pursuit of this promising approach.

It would be of interest to carry the ϵ-expansion fully to ϵ^4 and possibly farther, to ϵ^6, in connection with instability questions. The analysis to order ϵ^4 would have the important contribution of properly placing the $\nabla^4\alpha$ term in (35). Consideration of order ϵ^6 would permit description of travelling-wave instabilities that have been found by traditional analyses to occur in certain ranges of $L > 1$. It would also be of interest to remove the small-gradient approximation while retaining the small-intensity approximation to investigate regimes in which the classical two-zone structure of the wrinkled flame may become invalid. It could be of even greater interest to eliminate the small-intensity approximation but keep the small-gradient assumption with $\gamma \neq 0$ to investigate diffusive-thermal effects (beyond the kinematic effects derived in appendix A) on the dynamics of wrinkled flames in flows with large velocity fluctuations. In particular, it is known that sufficiently large flame stretch (strain rate) can produce local quenching of flamelets under suitable conditions even in the absence of heat loss. The nonlinearity would make the accurate inclusion of local quenching a difficult problem, but simplification may be achieved by addition of volumetric heat losses at a level sufficient to cause extinctions to occur in the regime of small stretch.

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Appendix A. Kinematic effects on flame motion

The effects summarized in (12) may be derived by allowing $\epsilon u_{-\infty}$ and $\epsilon \mathbf{v}_{-\infty}$ to be of order unity. Let u_α and $\mathbf{v}_\alpha$ denote respectively the values of these quantities at $X = \epsilon\alpha$. Then, with the time scaling of § 3, $\alpha_{0t} = u_\alpha$ results in α being large $O(1/\epsilon)$, while α_t and $\nabla\alpha$ remain $O(1)$. Only the dominant terms in the limit $\epsilon \to 0$ are to be considered here, i.e. effects of flame structure are ignored completely. With the adopted scaling, (2) shows that $\partial s/\partial\xi$ is $O(\epsilon)$, and therefore s may be treated as constant through the wrinkled laminar flame. By evaluating (1) just upstream from the flame, it is seen that this constant may be written as

$$s = m + u_\alpha - \alpha_t - \mathbf{v}_\alpha \cdot \nabla\alpha. \tag{A 1}$$

The only remaining conservation equations that need to be considered are (6) and (9), which to the dominant order in ϵ become

$$\left.\begin{aligned} s\,\partial\Theta/\partial\xi-(1+|\nabla\alpha|^2)\,\partial^2\Theta/\partial\xi^2 &= \Lambda_L F, \\ s\,\partial\Psi/\partial\xi-L^{-1}(1+|\nabla\alpha|^2)\,\partial^2\Psi/\partial\xi^2 &= -\Lambda_L F. \end{aligned}\right\} \tag{A 2}$$

In the limit $\beta\to\infty$, F is negligibly small except in a narrow region centred at $\xi=0$. Outside this zone for $\xi>0$, the bounded solutions are constants, independent of ξ, and therefore the boundary conditions require that $\Theta=1$ and $\Psi=0$ for $\xi>0$. Analysis of the narrow region reaction in the manner of Clavin & Williams (1979) or of Joulin & Clavin (1979) then provides jump conditions that result in boundary conditions at $\xi=0$ for the linear, homogeneous equations obtained from (A 2) outside the reaction zone in the range $\xi<0$, viz,

$$\Theta|_{\xi=0-}=1,\quad \Psi|_{\xi=0-}=0,\quad \partial\Theta/\partial\xi|_{\xi=0-}=(1+|\nabla\alpha|^2)^{-\frac{1}{2}}$$

and

$$\partial\Psi/\partial\xi|_{\xi=0-}=-L(1+|\nabla\alpha|^2)^{-\frac{1}{2}}.$$

The solutions to the homogeneous equations for $\xi<0$ that satisfy the upstream boundary conditions are $\Theta=C_1\exp[s\xi/(1+|\nabla\alpha|^2)]$ and $\Psi=1-C_2\exp[Ls\xi/(1+|\nabla\alpha|^2)]$, where C_1 and C_2 are constants. Applying the boundary conditions at $\xi=0$ to these solutions results in $s=(1+|\nabla\alpha|^2)^{\frac{1}{2}}$ and in

$$\Theta=\begin{cases}\exp[\xi/(1+|\nabla\alpha|^2)^{\frac{1}{2}}] & (\xi<0),\\ 1 & (\xi>0),\end{cases} \tag{A 3}$$

$$\Psi=\begin{cases}1-\exp[L\xi/(1+|\nabla\alpha|^2)^{\frac{1}{2}}] & (\xi<0),\\ 0 & (\xi>0),\end{cases} \tag{A 4}$$

for the normalized temperature and concentationr profiles in the moving frame. Use of this result for s in (A 1) then yields

$$\alpha_t-m=u_\alpha-\mathbf{v}_\alpha\,.\,\nabla\alpha-(1+|\nabla\alpha|^2)^{\frac{1}{2}}, \tag{A 5}$$

which has been quoted in (12).

Upstream from the wrinkled flame, it is evident from (8) that $r=1$; therefore, to the dominant order in ϵ, U may be written as $U=m+u(x,y,z,t)$ in this upstream region, where u is $O(1)$ and (although not explicitly indicated in the notation) varies only on the long time and distance scales. Continuity in this constant-density region is $u_x=-\nabla\,.\,\mathbf{v}$, where $\mathbf{v}(x,y,z,t)$ is the transverse velocity ahead of the flame. From this condition it may readily be shown that

$$u|_{x=\alpha}-\mathbf{v}|_{x=\alpha}\,.\,\nabla\alpha=u|_{x=0}-\nabla\,.\int_0^{\alpha(y,z,t)}\mathbf{v}(x',y,z,t)\,dx'. \tag{A 6}$$

In (A 5), clearly $u_\alpha=u|_{x=\alpha}$ and $\mathbf{v}_\alpha=\mathbf{v}_\alpha|_{x=\alpha}$, and therefore use of (A 6) on the right-hand side of (A 5) shows through homogeneity in the transverse directions, through $\overline{\alpha}=0$ and through $\overline{u}|_{x=0}=0$ that the average of (A 4) gives

$$m=\overline{(1+|\nabla\alpha|^2)^{\frac{1}{2}}}. \tag{A 7}$$

Equation (A 7) also has been quoted in (12).

Equation (A 5) constitutes a nonlinear evolution equation valid for arbitrary intensity of turbulence if curvature effects remain weak enough for the local structure of the wrinkled flame to be unmodified by turbulence. This is consistent with the fact that the solutions given by (A 3) and (A 4) are the laminar solutions with the independent variable $\xi/(1+|\nabla\alpha|^2)^{\frac{1}{2}}$. The geometrical stretching factor in the denominator here arises merely from the tilting of the local normal to the flame with respect to the direction of the mean flow.

The first term on the right-hand side of (A 5) describes a process in which the flame front follows the displacement field of the velocity fluctuations at its instantaneous location and is thus subject to a mechanism of turbulent diffusion, discussed by Clavin & Williams (1979). This diffusion process differs from that of the ordinary Lagrangian displacement in that it involves only the longitudinal component of velocity and in that the front is not fixed to a fluid element but instead traverses fluid elements at the constant laminar flame speed. Nevertheless, the streamwise Eulerian displacement is relevant to α only for small displacement of the flame front; for large displacements proper description of the motion involves more of a Lagrangian character.

The last two terms in (A 5) are nonlinear corrections associated with flame-front wrinkling. The $\mathbf{v}.\nabla\alpha$ term arises from transverse convection into a tilted flame element, and the last term describes the effect of the increase in flame area. Neither of these terms involves the transverse diffusion processes studied in the main text. The average of the last term is the average fractional increase in area of the flame front and therefore provides the geometrical influence on the turbulent flame speed (A 7), as discussed earlier (Clavin & Williams 1979, 1981).

Appendix B. Solution to second order without introduction of multiple-scale method

In place of (11), expansions may be adopted that do not involve use of the variable Ξ. The modified developments are

$$\left.\begin{aligned} U &= u_0(\xi)+\epsilon u_{-\infty}(X,Y,Z,T)+\epsilon u_1(\xi,Y,Z,T)+\dots,\\ \mathbf{v} &= \epsilon\mathbf{v}_{-\infty}(X,Y,Z,T)+\epsilon\mathbf{v}_1(\xi,Y,Z,T)+\dots,\\ p &= p_0(\xi)+\epsilon p_{-\infty}(X,Y,Z,T)+\epsilon p_1(\xi,Y,Z,T)+\dots,\\ \Theta &= \Theta_0(\xi)+\epsilon\Theta_1(\xi,Y,Z,T)+\dots,\\ \Psi &= \Psi_0(\xi)+\epsilon\Psi_1(\xi,Y,Z,T)+\dots, \end{aligned}\right\} \tag{B 1}$$

which are to be used along with the expansion

$$u_{-\infty}(X,Y,Z,T) = u_{-\infty}(0,Y,Z,T)+\epsilon(\xi+\alpha_0)\,u_{-\infty X}(0,Y,Z,T)+\dots, \tag{B 2}$$

and similar expansions for $\mathbf{v}_{-\infty}$ and $p_{-\infty}$. The dependences on Y, Z and T will no longer be exhibited explicitly in the notation.

Outside the reaction zone, to lowest order in ϵ (1)–(9) provide

$$\left.\begin{aligned} s_0 &= r_0 u_0, \quad ds_0/d\xi = 0,\\ s_0 du_0/d\xi &= -dp_0/d\xi + (P+P')\,d^2u_0/d\xi^2,\\ s_0 d\Theta_0/d\xi &= d^2\Theta_0/d\xi^2,\\ r_0 &= [1+\Theta_0\gamma/(1-\gamma)]^{-1},\\ s_0 d\Psi_0/d\xi &= L^{-1}d^2\Psi_0/d\xi^2, \end{aligned}\right\} \tag{B 3}$$

in the sequence shown. Since s_0 is thus independent of ξ, it is seen that

$$\left.\begin{aligned} &m_0 = s_0 = 1, \quad u_0 = 1/r_0,\\ &\Theta_0 = \Theta_{\mathrm{L}}(\xi) = \begin{cases} e^{\xi} & (\xi < 0),\\ 1 & (\xi > 0),\end{cases}\\ &\Psi_0 = \Psi_{\mathrm{L}}(\xi) = \begin{cases} 1-e^{L\xi} & (\xi < 0),\\ 0 & (\xi > 0),\end{cases}\\ &r_0 = \begin{cases} [1+e^{\xi}\gamma/(1-\gamma)]^{-1} & (\xi < 0),\\ 1-\gamma & (\xi > 0),\end{cases}\\ &p_0 = \begin{cases} (P+P'-1)\,e^{\xi}\gamma/(1-\gamma) & (\xi < 0),\\ -\gamma/(1-\gamma) & (\xi > 0).\end{cases} \end{aligned}\right\} \tag{B 4}$$

The results in (B 4) are derived in the sequence listed in (B 4) by use of (B 3), boundary conditions and jump conditions. Note that the normalized temperature and composition profiles possess the laminar solutions $\Theta_{\mathrm{L}}(\xi)$ and $\Psi_{\mathrm{L}}(\xi)$, in the moving frame. The solutions for r_0, u_0 and p_0 also correspond to the laminar solutions, there being a discontinuity in p_0 across the reaction zone to balance the discontinuity of the normal viscous stress. Since $m_0 = 1$, clearly $u_{\mathrm{T}} = u_{\mathrm{L}}$ to lowest order.

Collecting terms of order ϵ in (1)–(9) yields

$$\left.\begin{aligned} &s_1 = r_1 u_0 + r_0[u_{-\infty}(0) + u_1 - \alpha_{0\mathrm{T}}], \quad \partial s_1/\partial\xi = 0,\\ &\frac{\partial u_1}{\partial\xi} + s_1\frac{du_0}{d\xi} = -\frac{\partial p_1}{\partial\xi} + P\frac{\partial^2 u_1}{\partial\xi^2} + P'\frac{\partial^2}{\partial\xi^2}\left(\frac{s_1}{r_0} - \frac{r_1}{r_0^2}\right),\\ &\frac{\partial \mathbf{v}_1}{\partial\xi} = \hat{\nabla}\alpha_0\frac{dp_0}{d\xi} + \frac{\partial^2 u_1}{\partial\xi^2} + P'\frac{\partial^2}{\partial\xi^2}\left(\frac{s_1}{r_0} - \frac{r_1}{r_0^2}\right),\\ &\partial\Theta_1/\partial\xi + s_1 d\Theta_0/d\xi = \partial^2\Theta_1/\partial\xi^2, \quad r_1 = -r_0^2\Theta_1\gamma/(1-\gamma),\\ &\partial\Psi_1/\partial\xi + s_1 d\Psi_0/d\xi = L^{-1}\partial^2\Psi_1/\partial\xi^2. \end{aligned}\right\} \tag{B 5}$$

Here $\hat{\nabla}$ denotes the transverse gradient involving differentiation with respect to Y and Z, so that $\hat{\nabla}\alpha_0$ is of order unity. From (B 5), s_1 is independent of ξ, and therefore use of (1), (B 1), (B 2) and upstream boundary conditions shows that $s_1 = m_1 + u_{-\infty}(0) - \alpha_{0T}$. The equations for Θ_1 and Ψ_1 in (B 5) and their associated boundary conditions may be demonstrated to require that $s_1 = 0$ and then provide $\Theta_1 = 0$ and $\Psi_1 = 0$. Thus, $m_1 + u_{-\infty}(0) - \alpha_{0T} = 0$, the average of which shows that $m_1 = 0$. Therefore $\alpha_{0T} = u_{-\infty}(0)$, as anticipated. Since $\Theta_1 = 0$, (B 5) next yields $r_1 = 0$, then $u_1 = 0$, and hence p_1 is independent of ξ. The upstream boundary condition then shows that $p_1 = 0$. The only differential equation in (B 5) with a non-trivial solution is that for $\mathbf{v}_1$. By use of (B 4) and boundary and jump conditions, it may be shown that

$$\mathbf{v}_1 = (\hat{\nabla}\alpha_0)(1-u_0),$$

where a term giving exponential growth in ξ has been excluded. This $\mathbf{v}_1$ solution merely defines the transverse projection of the change in the normal velocity through the tilted laminar flame.

When terms of order ϵ^2 in (1)–(9) are collected and use is made of the results just derived, it is found that

$$\left.\begin{aligned}
&s_2 = r_2 u_0 + r_0[u_2 + (\xi+\alpha_0)\,u_{-\infty X}(0) - \alpha_{1T} - \mathbf{v}_{-\infty}(0)\,.\,\hat{\nabla}\alpha_0 + (u_0-1)\,|\hat{\nabla}\alpha_0|^2],\\
&\partial s_2/\partial\xi = (1-r_0)\,[\hat{\nabla}^2\alpha_0 + \hat{\nabla}\,.\,\mathbf{v}_{-\infty}(0)] - \hat{\nabla}\,.\,\mathbf{v}_{-\infty}(0),\\
&r_0 u_{-\infty T}(0) + u_{-\infty X}(0) + \frac{\partial u_2}{\partial\xi} + (s_2 + P\hat{\nabla}^2\alpha_0)\frac{du_0}{d\xi}\\
&\quad = -p_{-\infty X}(0) - \frac{\partial p_2}{\partial\nu} + P\left[\frac{\partial^2 u_2}{\partial\xi^2} + |\hat{\nabla}\alpha_0|^2\frac{d^2u_0}{d\xi^2}\right] + P'\left[\frac{\partial^2}{\partial\xi^2}\left(\frac{s_2}{r_0} - \frac{r_2}{r_0^2}\right) - \hat{\nabla}^2\alpha_0\frac{du_0}{d\xi}\right],\\
&r_0\mathbf{v}_{-\infty T}(0) - (1-r_0)\,\hat{\nabla}\alpha_{0T} + \partial\mathbf{v}_2/\partial\xi = -\hat{\nabla}p_{-\infty}(0) + P\frac{\partial^2\mathbf{v}_2}{\partial\xi^2} - P'\hat{\nabla}\alpha_1\frac{d^2u_0}{d\xi^2},\\
&\frac{\partial\Theta_2}{\partial\xi} + (s_2 + \hat{\nabla}^2\alpha_0)\frac{d\Theta_0}{d\xi} = \frac{\partial^2\Theta_2}{\partial\xi^2} + |\hat{\nabla}\alpha_0|^2\frac{d^2\Theta_0}{d\xi^2}, \quad r_2 = -r_0^2\Theta_2\frac{\gamma}{1-\gamma},\\
&\frac{\partial\Psi_2}{\partial\xi} + (s_2 + L^{-1}\hat{\nabla}^2\alpha_0)\frac{d\Psi_0}{d\xi} = L^{-1}\frac{\partial^2\Psi_2}{\partial\xi^2} + L^{-1}|\hat{\nabla}\alpha_0|^2\frac{d^2\Psi_0}{d\xi^2}.
\end{aligned}\right\} \tag{B 6}$$

Unlike (B 5), for which continuity of Θ_1, Ψ_1 and their first derivatives applied at $\xi = 0$, (B 6) must be solved subject to jump conditions on the first derivatives of Θ_2 and Ψ_2 at $\xi = 0$, obtained from the expansion of (13) in powers of ϵ. The second equation in (B 6) may be integrated, and the constant of integration may be evaluated from the boundary conditions by applying the first equation in (B 6) in the upstream flow. The formula

$$\begin{aligned}
s_2 = m_2 - \alpha_{1T} - \hat{\nabla}\,.\,[\alpha_0\mathbf{v}_{-\infty}(0)] + \xi u_{-\infty X}(0)\\
+ [\hat{\nabla}^2\alpha_0 - u_{-\infty X}(0)]\begin{cases}\ln\left(1 + \dfrac{\gamma}{1-\gamma}e^{\xi}\right) & (\xi \leqslant 0),\\[2ex] \ln\left(\dfrac{1}{1-\gamma}\right) & (\xi > 0)\end{cases}
\end{aligned} \tag{B 7}$$

is thereby obtained, where use has been made of the continuity expression

$$\nabla\,.\,\mathbf{v}_{-\infty} = -u_{-\infty X}.$$

Use of (B 4) and (B 7) in the last relationship of (B 6) yields

$$\begin{aligned}
\frac{\partial\Psi_2}{\partial\xi} - L^{-1}\frac{\partial^2\Psi_2}{\partial\xi^2} = \Big\{\alpha_{1T} - m_2 + \hat{\nabla}\,.\,[\alpha_0\mathbf{v}_{-\infty}(0)] - \xi u_{-\infty X}(0)\\
+ [u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0]\ln\left(1 + \frac{\gamma}{1-\gamma}e^{\xi}\right) - L^{-1}\hat{\nabla}^2\alpha_0 + |\hat{\nabla}\alpha_0|^2\Big\}\,Le^{L\xi}[H(\xi)-1].
\end{aligned} \tag{B 8}$$

The equation for Θ_2 obtained from (B 6) is the same as (B 8) except for a sign change of the right-hand side and the replacement of L by unity. These equations are to be solved subject to the vanishing of Ψ_2 and Θ_2 at $\xi = \pm\infty$ and to the jump conditions at $\xi = 0$ obtained from (13).

Up to this stage, the method is simpler than the use of multiple scales. However, the procedure for solving (B 8) and the corresponding equation for Θ_2 is more tedious. This procedure parallels that indicated by Clavin & Williams (1981). For $\xi > 0$, it is found that Ψ_2 and Θ_2 must remain constant to avoid exponential growth, whence boundary conditions require $\Psi_2^{(0)} = \Psi_2^{(1)} = \Theta_2^{(0)} = 0$ and $\Theta_2^{(1)} = \sigma_2$; the violation at first order in β^{-1} of the condition that Θ_2 vanish at $\xi = \infty$ is appropriate here because the adjustment of Θ_2 from $\beta^{-1}\sigma_2$ to zero occurs on the longer distance scale and need not be calculated (in fact cannot be calculated without introducing a two-scale treatment of Θ).

Although full solutions for $\xi < 0$ may be sought by standard techniques, the calculations are tedious, and the results are not needed; it is sufficient to obtain the relationships among the values and derivatives of the solutions at $\xi = 0$. This can be accomplished by integrating (B 8) and the equation for Θ_2 from $\xi = -\infty$ to $\xi = 0$ and using the boundary conditions at $\xi = -\infty$. For brevity write the quantity in the curly brackets on the right-hand side of (B 8) as

$$A + B\xi + C\ln\left[1 + \frac{\gamma}{1-\gamma}e^{\xi}\right],$$

where A, B and C are independent of ξ. Note that B and C are independent of L but that $A = A^{(0)} + \beta^{-1}lA^{(1)} + \ldots$, where $A^{(1)} = \hat{\nabla}^2\alpha_0$. The integration can be performed explicitly without expansion in β^{-1} for all terms except that involving C. In this term, the expansion $Le^{L\xi} = e^{\xi}[1 + \beta^{-1}l(1+\xi) + \ldots]$ is introduced prior to integration. Since Ψ_2 and $\partial\Psi_2/\partial\xi$ vanish at $\xi = -\infty$, the first integral of (B 8) then gives

$$L^{-1}\partial\Psi_2/\partial\xi|_{\xi=0-} - \Psi_2|_{\xi=0-} = A - L^{-1}B + C[F(\gamma) + \beta^{-1}lG(\gamma) + \ldots], \qquad \text{(B 9)}$$

where

$$F(\gamma) = \frac{1}{\gamma}\ln\left(\frac{1}{1-\gamma}\right) - 1 = \sum_{k=1}^{\infty}\frac{\gamma^k}{k+1}, \qquad \text{(B 10)}$$

$$G(\gamma) = \frac{1-\gamma}{\gamma}\int_0^{\gamma/(1-\gamma)}[1 - x^{-1}\ln(1+x)]\,dx$$
$$= \begin{cases} \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(k+1)^2}\left(\frac{\gamma}{1-\gamma}\right)^k & (\gamma < \tfrac{1}{2}), \\ 1 - \tfrac{1}{12}\pi^2 \quad (\gamma = \tfrac{1}{2}), & \\ 1 - \left(\dfrac{1-\gamma}{2\gamma}\right)\left[\ln\left(\dfrac{1}{1-\gamma}\right)\right]^2 + O(1-\gamma) & (\gamma \to 1). \end{cases} \qquad \text{(B 11)}$$

The first integral of the corresponding equation for Θ_2 provides

$$\partial\Theta_2/\partial\xi|_{\xi=0-} - \Theta_2|_{\xi=0-} = -A^{(0)} + B - CF(\gamma). \qquad \text{(B 12)}$$

The results in (B 9) and (B 12) may be expanded in powers of β^{-1} and then used in conjunction with the expansion in powers of ϵ of the conditions in (13) to obtain the desired information. The results from (13) that are needed are

$$\Theta_2^{(0)}|_{\xi=0-} = \Psi_2^{(0)}|_{\xi=0-} = \Psi_2^{(1)}|_{\xi=0-} = 0, \tag{B 13}$$

$$\partial\Theta_2^{(0)}/\partial\xi|_{\xi=0-} = \tfrac{1}{2}(\sigma_2 - |\hat{\nabla}\alpha_0|^2), \tag{B 14}$$

$$\partial\Theta_2^{(1)}/\partial\xi|_{\xi=0-} + \partial\Psi_2^{(1)}/\partial\xi|_{\xi=0-} = l\,\partial\Psi_2^{(0)}/\partial\xi|_{\xi=0-}; \tag{B 15}$$

the additional relationship $\partial\Theta_2^{(0)}/\partial\xi|_{\xi=0-} = -\partial\Psi_2^{(0)}/\partial\xi|_{\xi=0-}$ is satisfied already by (B 9) and (B 12). In view of (B 13), the term of order β^{-1} in the expansion of (B 9) is

$$\partial\Psi_2^{(1)}/\partial\xi|_{\xi=0-} - l\,\partial\Psi_2^{(0)}/\partial\xi|_{\xi=0-} = l[A^{(1)} + B + CG(\gamma)]. \tag{B 16}$$

From (B 15) and the term of order β^{-1} in the expansion of (B 12), this result provides

$$\partial\Theta_2^{(1)}/\partial\xi|_{\xi=0-} = -l[A^{(1)} + B + CG(\gamma)] = \Theta_2^{(1)}|_{\xi=0-}. \tag{B 17}$$

Use of (B 13) and (B 14) in the term of order unity in (B 12) shows that

$$\tfrac{1}{2}(\sigma_2 - |\hat{\nabla}\alpha_0|^2) = -A^{(0)} + B - CF(\gamma). \tag{B 18}$$

In view of the definition $\Theta_2^{(1)}|_{\xi=0-} = \sigma_2$, the last relationship in (B 17) may be written as

$$\sigma_2 = -l[A^{(1)} + B + CG(\gamma)], \tag{B 19}$$

and may then be substituted into (B 18) to provide an expression for $A^{(0)}$.

From the definitions of A, B and C, it is seen that (B 19) states that

$$\sigma_2 = l[u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0][1 - G(\gamma)]. \tag{B 20}$$

Substitution into (B 18) then shows that

$$\alpha_{1T} = m_2 - \tfrac{1}{2}|\hat{\nabla}\alpha_0|^2 - \hat{\nabla}\,.\,[\alpha_0\mathbf{v}_{-\infty}(0)] - [u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0]\{[1 + F(\gamma)] + \tfrac{1}{2}l[1 - G(\gamma)]\}. \tag{B 21}$$

Use of homogeneity, $\overline{u}_{-\infty} = 0$ and $\overline{\alpha}_1 = 0$ in the average of (B 21) provides the first non-vanishing correction for the turbulent flame speed,

$$m_2 = \tfrac{1}{2}\overline{|\hat{\nabla}\alpha_0|^2}. \tag{B 22}$$

Appendix C. Fourth-order solution

Attention is now restricted to the limit $\gamma \to 0$. Put $U = m + \epsilon u_{-\infty}$ and $\mathbf{v} = \epsilon\mathbf{v}_{-\infty}$, and treat $u_{-\infty}$ and $\mathbf{v}_{-\infty}$ as known functions. Here $r = 1$, $s = m + \epsilon[u_{-\infty} - \alpha_T - \epsilon\mathbf{v}_{-\infty}\,.\,\hat{\nabla}\alpha]$, and only the solutions to (6) and (9) need to be sought. When terms of order ϵ^3 are collected in (9) and an average is taken, it is found that

$$\frac{\partial\overline{\Psi}_3}{\partial\xi} - L^{-1}\frac{\partial^2\overline{\Psi}_3}{\partial\xi^2} = 2\overline{\hat{\nabla}\alpha_0\,.\,\hat{\nabla}\alpha_1}\,L^{-1}\frac{d^2\Psi_0}{d\xi^2} - m_3\frac{d\Psi_0}{d\xi}. \tag{C 1}$$

A similar equation for $\overline{\Theta}_3$ is obtained from the average of the expansion of (6). Use of terms of order ϵ^3 obtained from the jump conditions (13) then leads to the solutions $\overline{\Psi}_3 = \overline{\Theta}_3 = 0$ for $\xi > 0$, to $\overline{\Theta}_3$ and $\overline{\Psi}_3$ being functions that can be obtained from expansions of (A 3) and (A 4) for $\xi < 0$, and to

$$m_3 = \overline{\hat{\nabla}\alpha_0\,.\,\hat{\nabla}\alpha_1}. \tag{C 2}$$

When terms of order ϵ^4 are collected in (9) and an average is taken, it is found after manipulation that

$$\frac{\partial \overline{\Psi}_4}{\partial \xi} - L^{-1}\frac{\partial^2 \overline{\Psi}_4}{\partial \xi^2} = \overline{[|\hat{\nabla}\alpha_1|^2 + 2\hat{\nabla}\alpha_0 \,.\, \hat{\nabla}\alpha_2]}\, L^{-1}\frac{d^2\Psi_0}{d\xi^2} - m_4 \frac{d\Psi_0}{d\xi} - \overline{u_{-\infty X}(0)\frac{\partial(\xi\Psi_2)}{\partial \xi}}$$
$$+ \overline{[(L^{-1}+1+\tfrac{1}{2}l)\,\hat{\nabla}^2\alpha_0 - \tfrac{1}{2}|\hat{\nabla}\alpha_0|^2 - (1+\tfrac{1}{2}l)\,u_{-\infty X}(0)]\frac{\partial \Psi_2}{\partial \xi}}$$
$$+ \overline{L^{-1}|\hat{\nabla}\alpha_0|^2 \frac{\partial^2 \Psi_2}{\partial \xi^2}}. \qquad \text{(C 3)}$$

The equation for $\overline{\Theta}_4$, derived from (6), may be obtained from (C 3) by replacing Ψ by Θ and L by unity (but not replacing l by zero). These equations are integrated from $\xi = -\infty$ to $\xi = 0$ in the manner of appendix B, with results of appendix B for $F(\gamma) = G(\gamma) = 0$ used to obtain values and derivatives of the second-order solutions at $\xi = 0$. The averages of the terms of order ϵ^4 in the expansion of the jump conditions (13) are then used along with the results of the integration, expanded to two terms in β^{-1}, to provide a set of simultaneous algebraic equations that can be solved for $\overline{\sigma}_4$ and m_4.

In pursuing these computations it must be recognized that although $\Psi_4 = 0$ and $\Theta_4^{(0)} = 0$ for $\xi > 0$, the non-zero value of $\Theta_2^{(1)}$ for $\xi > 0$, obtained in appendix B, requires that

$$\overline{\Theta_4^{(1)}} = \overline{\Theta_4^{(1)}}|_{\xi=0} - \overline{u_{-\infty X}(0)\,\sigma_2}\,\xi, \qquad \text{(C 4)}$$

for $\xi > 0$; a two-scale analysis is needed if the boundary condition for Θ_4 at $\xi = \infty$ is to be satisfied to order β^{-1}. The linear term in ξ in (C 4) is a consequence of the Θ-analogue of $-\overline{u_{-\infty X}(0)\,\partial(\xi\Psi_2)/\partial\xi}$ in (C 3), and arises originally from $v\,\partial\Theta/\partial\eta + w\,\partial\Theta/\partial\zeta$ in (6).

Solution of the algebraic equations yields

$$\overline{\sigma}_4 = -l\overline{[u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0]\{(2+\tfrac{1}{2}l)\,[u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0] + \tfrac{1}{2}|\hat{\nabla}\alpha_0|^2\}}, \qquad \text{(C 5)}$$

and

$$m_4 = -l(1+\tfrac{1}{8}l)\,\overline{[u_{-\infty X}(0) - \hat{\nabla}^2\alpha_0]^2} + \overline{\hat{\nabla}\alpha_0 \,.\, \hat{\nabla}\alpha_2} + \tfrac{1}{2}\overline{|\hat{\nabla}\alpha_1|^2} - \tfrac{1}{8}\overline{|\hat{\nabla}\alpha_0|^4}. \qquad \text{(C 6)}$$

The result in (C 6) has been employed in writing (33).

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